Univerzitet u Zenici
Ekonomski fakultet

Zadaci sa pismenog ispita rađenog 11.02.2009. iz predmeta **Matematika**, sve četri grupe

1. Izračunati x ako se zna da je u binomnom razvoju $(\frac{\sqrt[6]{2}}{\sqrt[x]{3}} + \sqrt[x]{3})^{11}$ šesti član jednak 2772.

2. Matematičkom indukcijom dokazati da jednakost vrijedi za sve prirodne brojeve.

$$1^2 + 4^2 + 7^2 + ... + (3n-2)^2 = \frac{n(6n^2-3n-1)}{2} \ .$$

3. Diskutovati rang matrice $M = \begin{bmatrix} 1 & 10 & -6 & \lambda \\ 2 & -1 & \lambda & 3 \\ 1 & \lambda & -1 & 2 \end{bmatrix}$.

4. Riješiti matričnu jednačinu $A^{-1}X = I + BX$, $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 \\ -3 & 3 \end{bmatrix}$.

5. Ispitati funkciju i nacrtati graf: $y = \dfrac{16}{x^3 - 4x^2}$.

6. Ispitati i nacrtati funkciju: $y = \dfrac{x^3}{x^2 - 2x - 8}$.

7. Ispitati funkciju i nacrtati graf: $y = (x-1)e^{\frac{-1}{x+1}}$.

8. Ispitati funkciju i nacrtati graf: $y = ln\dfrac{x^2+3}{x^2+1}$.

9. Izračunati integral $I = \int x^3 e^{3x}\, dx$.

10. Izračunati integral $I = \int \dfrac{3-x}{2x^2+2x+1}\, dx$.

11. Izračunati površinu površi ograničenog krivom $y = x^2 - 4x + 3$ i pravama $y = 0$, $x = 0$, $x = 2$.

12. Izračunati površinu površi koja se nalazi u prvom kvadrantu, a ograničena je hiperbolom $xy = 4$ i parabolom $y = x^2 + x + 4$.

13. Naći ekstreme funkcije $z = e^{x^2-y}(5 - 2x + y)$.

14. Naći uslovne ekstreme funkcije $z = x^2 + xy + y^2$, ako je $4x^2 + 4xy + y^2 = 1$.

15. Riješiti diferencijalnu jednačinu $(x + y - 2)dx + (x - y + 4)dy = 0$.

16. Riješiti diferencijalnu jednačinu $x(2 + x)y' + 2(1 + x)y = 1 + 3x^2$, uz početni uslov $y(-1) = 1$.

Pismeni ispit iz Matematike, rađen 11.02.2009.
Neki zadaci nisu detaljno urađeni.
Za uočene greške pisati na infoarrt@gmail.com

1. Izračunati x ako se zna da je u binomnom razvoju $\left(\frac{\sqrt[6]{2}}{\sqrt[x]{3}}+\sqrt[x]{3}\right)^{11}$ šesti član jednak 2772.

Rj.
$$\left(\frac{\sqrt[6]{2}}{\sqrt[x]{3}}+\sqrt[x]{3}\right)^{11}=\sum_{k=0}^{11}\binom{11}{k}\left(2^{\frac{1}{6}}\cdot 3^{-\frac{1}{x}}\right)^{11-k}\cdot\left(3^{\frac{1}{x}}\right)^{k}=$$
$$=\sum_{k=0}^{11}\binom{11}{k}2^{\frac{11-k}{6}}\cdot 3^{-\frac{11-k}{x}+\frac{k}{x}}=\sum_{k=0}^{11}\binom{11}{k}2^{\frac{11-k}{6}}3^{\frac{-11+2k}{x}}$$

za $k=5$ imamo 6 član

$$\binom{11}{5}2\cdot 3^{-\frac{1}{x}}=2772 \quad /:2$$
$$\frac{11\cdot 10\cdot 9\cdot 8\cdot 7}{2\cdot 3\cdot 4\cdot 5}\cdot 3^{-\frac{1}{x}}=1386$$

$$11\cdot 3^{-\frac{1}{x}}=33 \quad /:11$$
$$3^{-\frac{1}{x}}=3$$
$$x=-1$$

za $x=-1$ šesti član u razvoju binoma je jednak 2772.

2. Dokazati matematičkom indukcijom tvrdnju
$$1^2+4^2+7^2+\dots+(3n-2)^2=\frac{n(6n^2-3n-1)}{2} \quad \text{gdje je } n\in\mathbb{N}.$$

Rj.
$$1^2+4^2+\dots+(3k-2)^2=\frac{k(6k^2-3k-1)}{2},\quad k\in\mathbb{N}.$$

BAZA INDUKCIJE

za $k=1$ imamo $1^2=\frac{1(6\cdot 1^2-3\cdot 1-1)}{2}$ tj. $1=1$

Jednakost je tačna za $k=1$.

KORAK INDUKCIJE

pretpostavimo da je jednakost tačna za $k=1,2,\dots,n$.
dokažimo da jednakst je tačna za $n+1$. Imamo
$$1^2+4^2+\dots+(3n-2)^2+(3(n+1)-2)^2=\frac{(n+1)(6(n+1)^2-3(n+1)-1)}{2}$$

$$\underbrace{1^2+4^2+\ldots+(3n-2)^2}_{\text{prema pretpostavci}}+(3n+2)^2=\frac{(n+1)(6n^2+9n+2)}{2}$$

$$\frac{n(6n^2-3n-1)}{2}+(3n+1)^2=\frac{(n+1)(6n^2+9n+2)}{2}$$

$$\frac{6n^3+15n^2+11n+2}{2}=\frac{(n+1)(6n^2+9n+2)}{2}$$

$$(6n^3+15n^2+11n+2):(n+1)=6n^2+9n+2$$

$$\begin{array}{l}
-\ 6n^3+6n^2 \\
\hline
\quad 9n^2+11n+2 \\
\quad -\ 9n^2+9n \\
\hline
\qquad 2n+2 \\
\qquad 2n+2 \\
\hline
\qquad = \quad =
\end{array}$$

imamo:

$$\frac{(n+1)(6n^2+9n+2)}{2}=\frac{(n+1)(6n^2+9n+2)}{2}$$

jednakost je tačna za $n+1$

ZAKLJUČAK

Jednakost je tačna za svaki prirodan broj.

3. Diskutovati rang matrice $M=\begin{bmatrix}1 & 10 & -6 & \lambda \\ 2 & -1 & \lambda & 3 \\ 1 & \lambda & -1 & 2\end{bmatrix}$ za razne vrijednosti parametra.

Rj.

$$M=\begin{bmatrix}1 & 10 & -6 & \lambda \\ 2 & -1 & \lambda & 3 \\ 1 & \lambda & -1 & 2\end{bmatrix}\underset{III_v-I_v}{\overset{II_v-2\cdot I_v}{\sim}}\begin{bmatrix}1 & 10 & -6 & \lambda \\ 0 & -21 & \lambda+12 & 3-2\lambda \\ 0 & \lambda-10 & 5 & 2-\lambda\end{bmatrix}\overset{II_v\leftrightarrow III_v}{\sim}$$

$$\begin{bmatrix}1 & 10 & -6 & \lambda \\ 0 & \lambda-10 & 5 & 2-\lambda \\ 0 & -21 & \lambda+12 & 3-2\lambda\end{bmatrix}\underset{\lambda\neq 10}{\overset{III_v-II_v\cdot\frac{21}{\lambda-10}}{\sim}}\begin{bmatrix}1 & 10 & -6 & \lambda \\ 0 & \lambda-10 & 5 & 2-\lambda \\ 0 & 0 & \frac{(\lambda+5)(\lambda-3)}{(\lambda-10)} & \frac{(\lambda-3)(\lambda+2)}{\lambda-10}\end{bmatrix}$$

Za $\lambda=10$ imamo

$$M=\begin{bmatrix}1 & 10 & -6 & 10 \\ 2 & -1 & 10 & 3 \\ 1 & 10 & -1 & 2\end{bmatrix}\underset{II_v-I_v\cdot 2}{\overset{III_v-I_v}{\sim}}\begin{bmatrix}1 & 10 & -6 & 10 \\ 0 & -21 & 22 & -17 \\ 0 & 0 & 5 & -8\end{bmatrix}$$

Diskusija:

1° $\lambda=3$ $\quad rang(M)=2$

2° $\lambda\neq 3$ $\quad rang(M)=3$

4. Riješiti matričnu jednačinu $A^{-1}X = I + BX$ gdje su $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -2 \\ -3 & 3 \end{bmatrix}$.

Rj.

$$A^{-1}X = I + BX$$

$$A^{-1}X - BX = I$$

$$\underbrace{(A^{-1} - B)}_{C} X = I$$

$$CX = I \quad / \cdot C^{-1} \text{ sa lijeve strane}$$

$$X = C^{-1}$$

$$\det A = \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix} = 1$$

$$A^{-1} = \frac{1}{\det A} \cdot A_{adj}$$

$$A_{adj} = [A_{kof}]^T$$

$$A^{-1} = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} - B = C = \begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -2 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$$

$$C^{-1} = \frac{1}{\det C} \cdot C_{adj}$$

$$\det C = \begin{vmatrix} 5 & 0 \\ 0 & 2 \end{vmatrix} = 10$$

$$C_{kof} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

$$c_{11} = (-1)^2 \cdot |2| = 2$$

$$c_{12} = (-1)^3 \cdot |0| = 0$$

$$c_{21} = (-1)^3 |0| = 0$$

$$c_{22} = (-1)^4 \cdot |5| = 5$$

$$C_{kof} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$$

$$C_{adj} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

traženo rješenje

5. Ispitati f-ju i nacrtati graf: $y = \dfrac{16}{x^3 - 4x^2}$.

Rj. $y = \dfrac{16}{x^2(x-4)}$

f-ja nema nule i ne siječe Y osu

$D: (-\infty, 0) \cup (0,4) \cup (4, +\infty)$

x	$(-\infty, 0)$	$(0,4)$	$(4,+\infty)$
y	$-$	$-$	$+$

znak f-je

f-ja nije ni parna ni neparna
nije periodična

$\lim_{x \to 0-} f(x) = -\infty \Rightarrow x=0$ je vert. asimp.

$\lim_{x \to 0+} f(x) = -\infty \Rightarrow x=0$ je vert. asim.

$\lim_{x \to 4+0} f(x) = +\infty \Rightarrow$ $x=4$ je vert. asim.

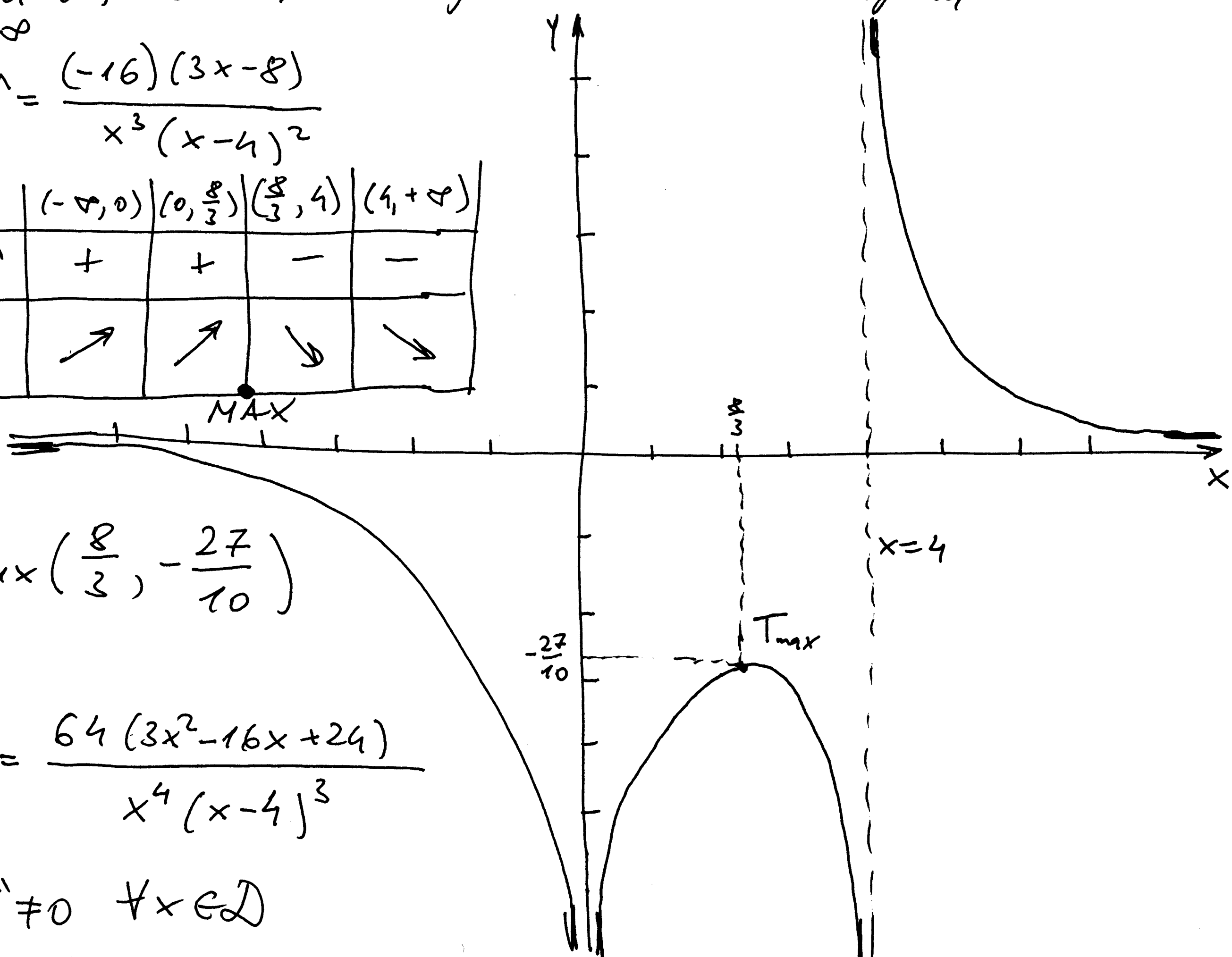
$\lim_{x \to 4-0} f(x) = -\infty \Rightarrow$ $x=4$ je vert. asim.

$\lim_{x \to \pm\infty} f(x) = 0 \Rightarrow y=0$ je horizont. asimptota

$y' = \dfrac{(-16)(3x-8)}{x^3(x-4)^2}$

x	$(-\infty, 0)$	$(0, \frac{8}{3})$	$(\frac{8}{3}, 4)$	$(4, +\infty)$
y'	$+$	$+$	$-$	$-$
y	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$

MAX



$T_{max}\left(\dfrac{8}{3}, -\dfrac{27}{10}\right)$

$y'' = \dfrac{64(3x^2 - 16x + 24)}{x^4(x-4)^3}$

$y'' \neq 0 \quad \forall x \in D$

f-ja nema prevojnih tački

x	$(-\infty, 0)$	$(0,4)$	$(4,+\infty)$
y''	$-$	$-$	$+$
y	$\frown$	$\frown$	$\smile$

60) Ispitati i nacrtati f-ju $y = \frac{x^3}{x^2-2x-8}$.

Rj. $y = \frac{x^3}{x^2-2x-8} = \frac{x^3}{(x+4)(x-2)}$

$\mathcal{D}: x \in (-\infty,-2) \cup (-2,4) \cup (4,+\infty)$

f-ja nije ni parna ni neparna

nije periodična

(0,0) je nula f-je i presjek sa y-osom

x	$(-\infty,-2)$	$(-2,0)$	$(0,4)$	$(4,+\infty)$
x^3	$-$	$-$	$+$	$+$
$x+4$	$-$	$-$	$-$	$+$
$x-2$	$-$	$+$	$+$	$+$
y	$-$	$+$	$-$	$+$

Znak f-je

$\lim_{x\to -2-0} f(x) = -\infty \Rightarrow x=-2$ je vert. asimp.

$\lim_{x\to -2+0} f(x) = +\infty \Rightarrow x=-2$ je vert. asimpt.

$\lim_{x\to 4-0} f(x) = -\infty \Rightarrow x=4$ je vert. asimpt.

$\lim_{x\to 4+0} f(x) = +\infty \Rightarrow x=4$ je vert. asimp.

$\lim_{x\to \pm\infty} f(x) = \pm\infty$ nema horiz. asimpt.

$y = kx+n$

$k = \lim_{x\to\infty} \frac{f(x)}{x} = 1 \quad n = \lim[f(x) - k\cdot x] = 2$

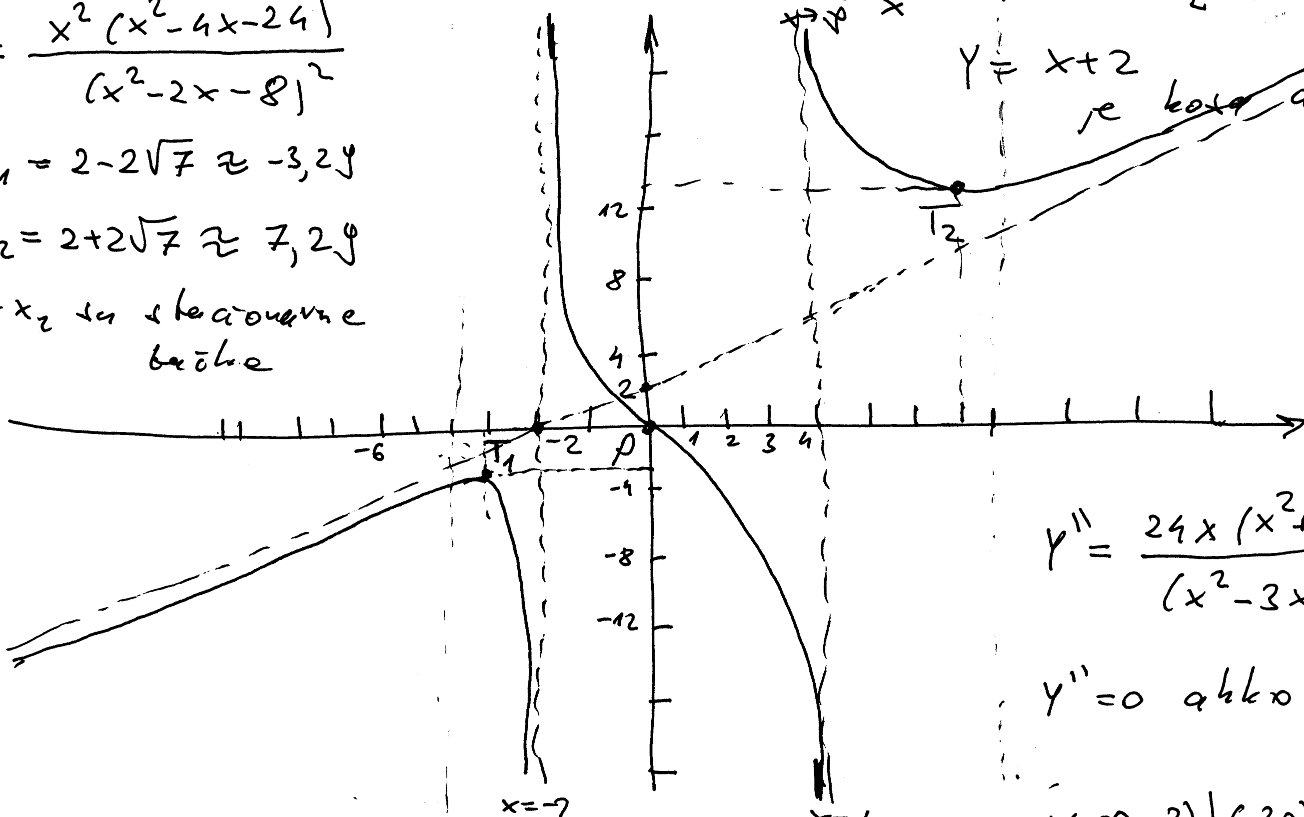
$y = x+2$ je kosa asimptota

$y' = \frac{x^2(x^2-4x-24)}{(x^2-2x-8)^2}$

$x_1 = 2-2\sqrt{7} \approx -3,29$

$x_2 = 2+2\sqrt{7} \approx 7,29$

x_1 i x_2 su stacionarne tačke



$y'' = \frac{24x(x^2+4x+16)}{(x^2-3x-8)^3}$

$y''=0$ akko $x=0$

x	$(-\infty,-4)$	$(-4,-3,29)$	$(-3,29,\ 2)$	$(2,\ 7,29)$	$(7,29,+\infty)$
y'	$+$	$+$	$-$	$-$	$+$
y	↗	↗	↘	↘	↗

max $T_1(-3,29,\ -3,78)$ min $T_2(7,29,\ 12,67)$

x	$(-\infty,-2)$	$(-2,0)$	$(0,4)$	$(4,+\infty)$
y''	$-$	$+$	$-$	$+$
y	$\frown$	$\cup$	$\cap$	$\cup$

P.T.

P(0,0) je prevojna tačka

7. Ispitati f-ju i nacrtati graf $y=(x-1)e^{\frac{-1}{x+1}}$.

Rj: $y=(x-1)e^{\frac{-1}{x+1}}$

$e^x > 0 \quad \forall x \in \mathbb{R}$

$D: x \in (-\infty,-1) \cup (-1,+\infty)$

f-ja nije ni parna ni neparna
nije periodična

$(1,0)$ nula f-je

$(0,-\frac{1}{e})$ presjek sa Y-osom

x	$(-\infty,-1)$	$(-1,1)$	$(1,+\infty)$
y	$-$	$-$	$+$

znak f-je

$\lim\limits_{x\to -1-0} f(x) = -\infty \Rightarrow x=-1$ je vertikalna asimpt.

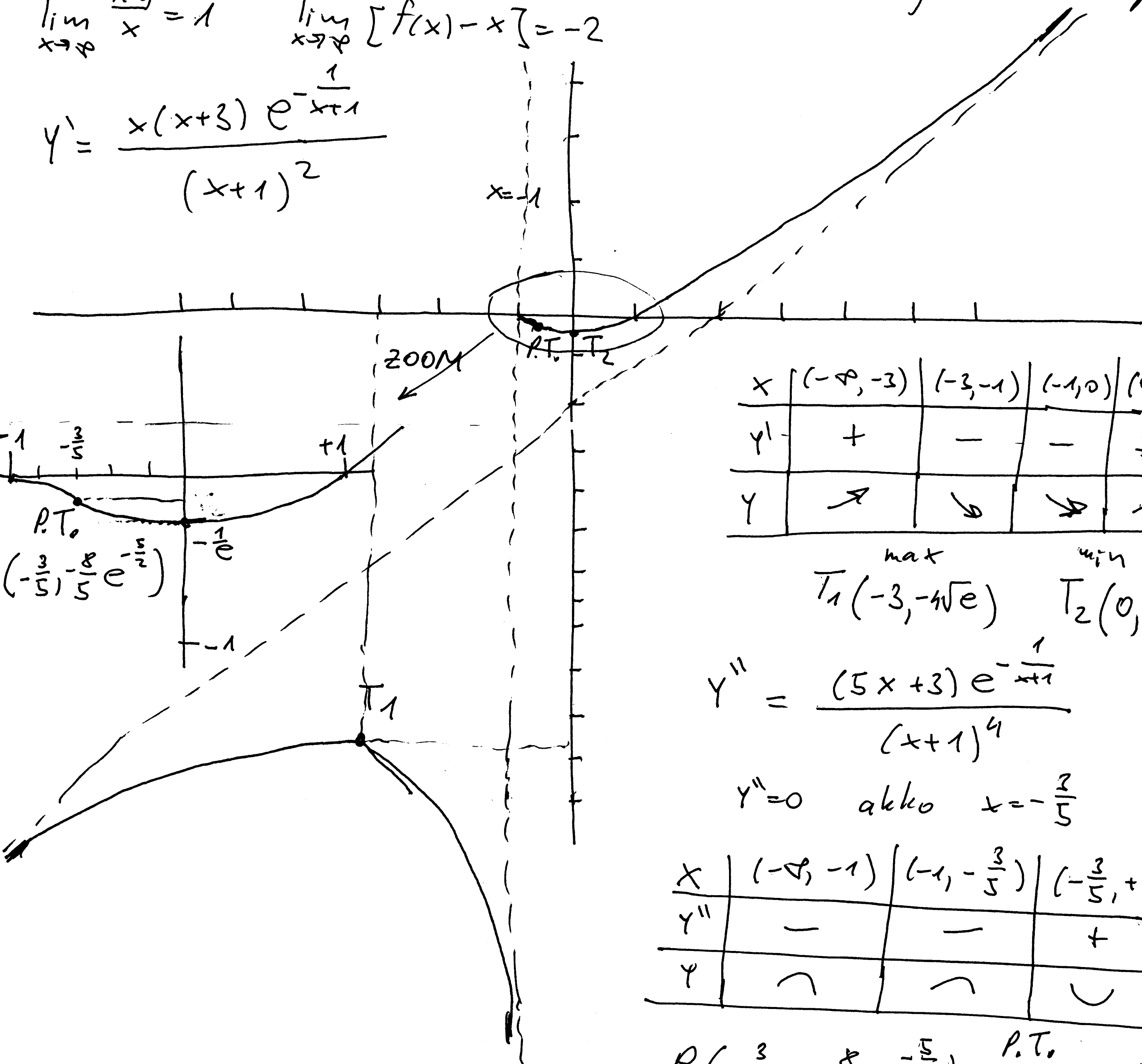
$\lim\limits_{x\to -1+0} f(x) = 0$

$\lim\limits_{x\to\pm\infty} f(x) = \pm\infty \Rightarrow$ f-ja nema horizont. asimpt.

$\lim\limits_{x\to\infty} \frac{f(x)}{x} = 1 \qquad \lim\limits_{x\to\infty}[f(x)-x] = -2$

$y = x-2$ je kosa asimptota

$$y' = \frac{x(x+3)e^{-\frac{1}{x+1}}}{(x+1)^2}$$



x	$(-\infty,-3)$	$(-3,-1)$	$(-1,0)$	$(0,+\infty)$
y'	$+$	$-$	$-$	$+$
y	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$

max $T_1(-3,-4\sqrt{e})$ min $T_2(0,-\frac{1}{e})$

$$y'' = \frac{(5x+3)e^{-\frac{1}{x+1}}}{(x+1)^4}$$

$y''=0$ akko $x=-\frac{3}{5}$

x	$(-\infty,-1)$	$(-1,-\frac{3}{5})$	$(-\frac{3}{5},+\infty)$
y''	$-$	$-$	$+$
y	$\frown$	$\frown$	$\smile$

$P(-\frac{3}{5}, -\frac{8}{5}e^{-\frac{5}{2}})$ P.T. prevojne tačke

8. Ispitati f-ju i nacrtati graf $y = \ln \frac{x^2+3}{x^2+1}$.

Rj: $D: x \in \mathbb{R}$

f-ja je parna

nije periodična

$f(x) > 0 \quad \forall x \in \mathbb{R}$

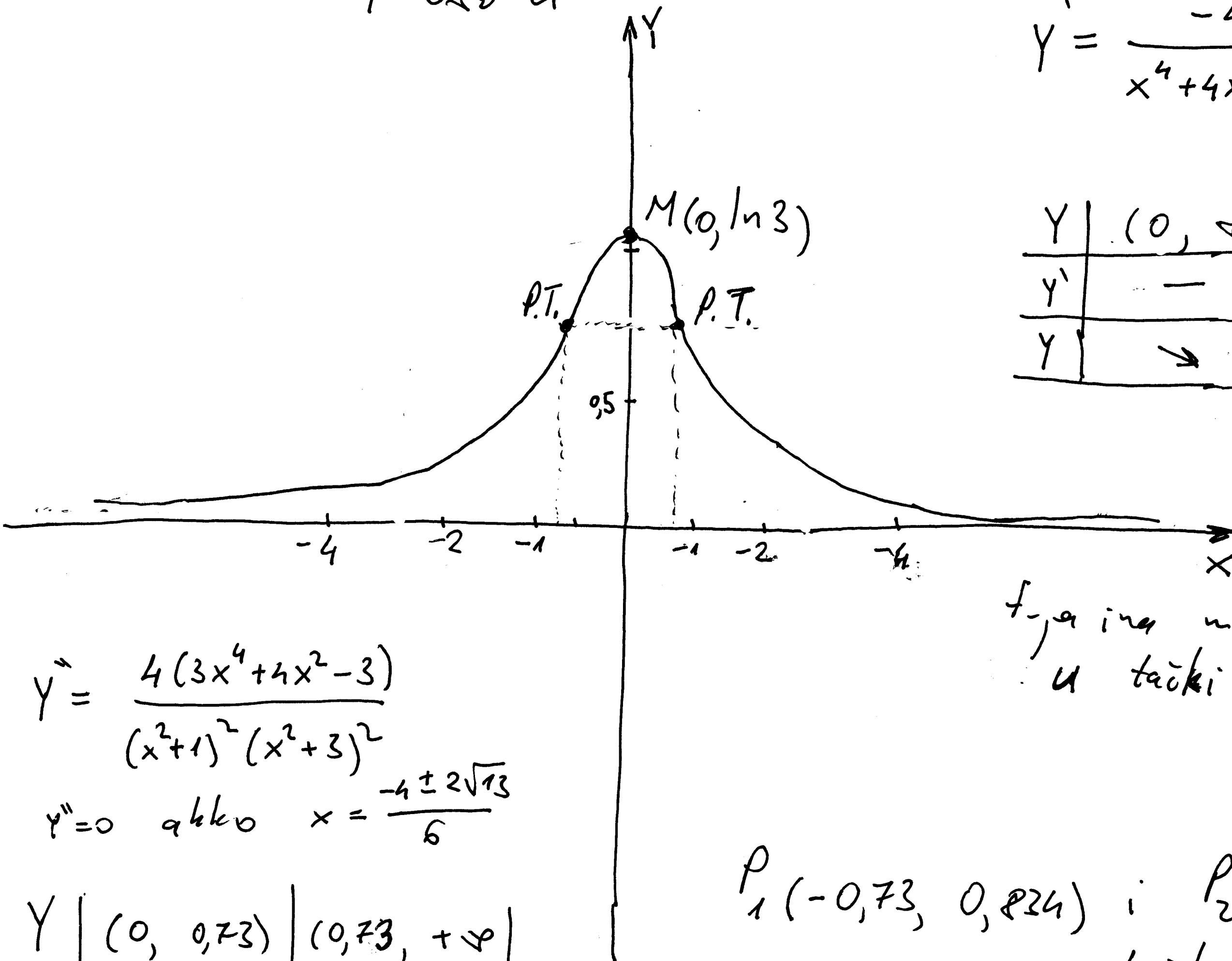
(f-ja nema nule)

$(0, \ln 3)$ tačka presjeka sa y-osom

f-ja nema vertikalne asimptote

$\lim_{x \to -\infty} f(x) = 0 \Rightarrow y=0$ je horizont. asi

$\lim_{x \to \infty} f(x) = 0 \Rightarrow y=0$ je horiz. asimpt.



$$y' = \frac{-4x}{x^4+4x^2+3}$$

y	$(0, \infty)$
y'	$-$
y	$\searrow$

$\Downarrow$

f-ja ima maksimum u tački $M(0, \ln 3)$

$$y'' = \frac{4(3x^4+4x^2-3)}{(x^2+1)^2(x^2+3)^2}$$

$y''=0$ akko $x = \frac{-4 \pm 2\sqrt{13}}{6}$

y	$(0,\ 0{,}73)$	$(0{,}73,\ +\infty)$
y''	$-$	$+$
y	$\frown$	$\smile$

P. T.

$P_1(-0{,}73,\ 0{,}834)$ i $P_2(0{,}73,\ 0{,}834)$ su prevojne tačke f-je

9. Izračunati integral $I=\int x^2 e^{3x} dx$.

Rj.

$$I=\int x^2 e^{3x}\,dx = \left| \begin{array}{ll} u=x^2 & dv=e^{3x}dx \\ du=2x\,dx & v=\frac{1}{3}e^{3x} \end{array} \right| = \frac{1}{3}x^2 e^{3x} - \frac{2}{3}\int x e^{3x}\,dx$$

$$\int x e^{3x}\,dx = \left| \begin{array}{ll} u=x & dv=e^{3x}dx \\ du=dx & v=\frac{1}{3}e^{3x} \end{array} \right| = \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}dx = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

$$I = \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$$ rješenje integrala

10. Izračunati integral $I=\int \frac{3-x}{2x^2+2x+1}\,dx$.

Rj.

$$I = -\frac{1}{4}\int \frac{4x-12}{2x^2+2x+1}\,dx = -\frac{1}{4}\int \frac{4x+2-14}{2x^2+2x+1}\,dx =$$

$$= -\frac{1}{4}\int \frac{4x+2}{2x^2+2x+1}\,dx + \frac{14}{4}\int \frac{dx}{2x^2+2x+1}$$

$$\int \frac{dx}{2x^2+2x+1} = \frac{1}{2}\int \frac{dx}{x^2+x+\frac{1}{2}} = \frac{1}{2}\int \frac{dx}{(x+\frac{1}{2})^2+\frac{1}{4}} = \left| \begin{array}{l} x+\frac{1}{2}=\frac{1}{2}t \\ dx=\frac{1}{2}dt \end{array} \right|$$

$$= \frac{1}{2}\cdot\frac{1}{2}\int \frac{dt}{\frac{1}{4}t^2+\frac{1}{4}} = \int \frac{dt}{t^2+1} = \operatorname{arc\,tg} t + C = \operatorname{arc\,tg}(2x+1) + C$$

$$I = -\frac{1}{4}\ln|2x^2+2x+1 + \frac{7}{2}\operatorname{arc\,tg}(2x+1) + C$$

11. Izračunati površinu lika ograničenog krivom $y = x^2 - 4x + 3$ i pravama $y = 0$, $x = 0$, $x = 2$.

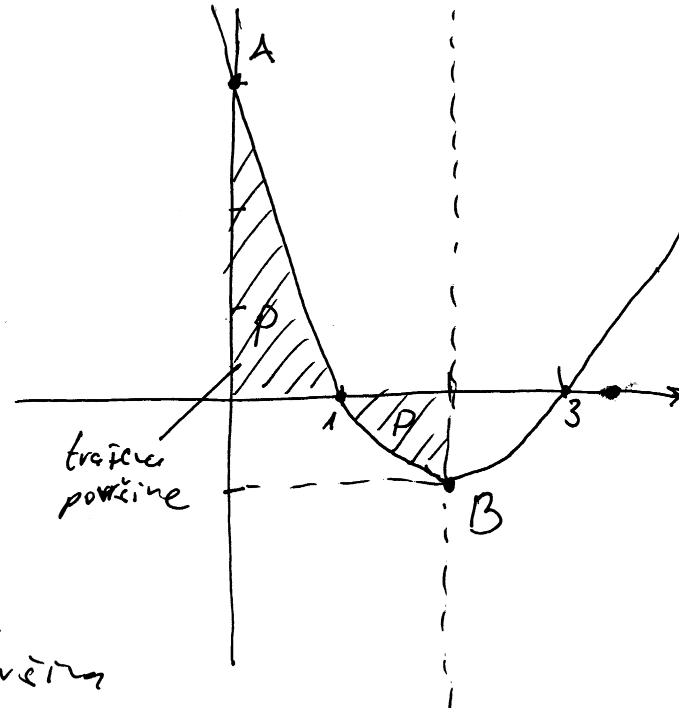
Rj. $y = x^2 - 4x + 3$ je parabola čije su nule 1 i 3.

$T(2, -1)$ je tjeme parabole $\left(T\left(-\frac{b}{2a}, -\frac{D}{4a}\right)\right)$.

$A(0,3)$ je tačka presjeka parabole i y ose

$$\begin{array}{l} y = x^2 - 4x + 3 \\ x = 2 \\ \hline \end{array}$$

$B(2, -1)$ je tačka presjeka parabole i y-ose



$$P = \int_0^1 (x^2 - 4x + 3)\,dx - \int_1^2 (x^2 - 4x + 3)\,dx$$

$$= \frac{4}{3} - \left(-\frac{2}{3}\right) = 2$$ tražena površina

12. Izračunati površinu površi koji se nalazi u I kvadrantu, a ograničen je hiperbolom $xy = 4$ i parabolom $y = -x^2 + x + 4$.

Rj. $y = -x^2 + x + 4$ je parabola sa nulama $x_1 = \frac{1+\sqrt{17}}{2} \approx 2{,}56$ i $x_2 = \frac{-1+\sqrt{17}}{-2} \approx -1{,}56$

$T\left(\frac{1}{2}, \frac{\sqrt{17}}{4}\right)$ je tjeme parabole

$A(0,4)$ je tačka presjeka parabole i y-ose

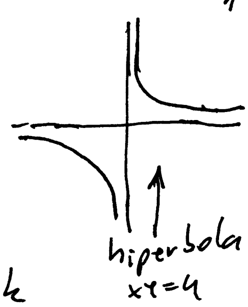
$$\begin{array}{l} y = -x^2 + x + 4 \\ xy = 4 \\ \hline -x^3 + x^2 + 4x - 4 = 0 \end{array}$$

za $x = 0$: $\quad -4 = 0$ /

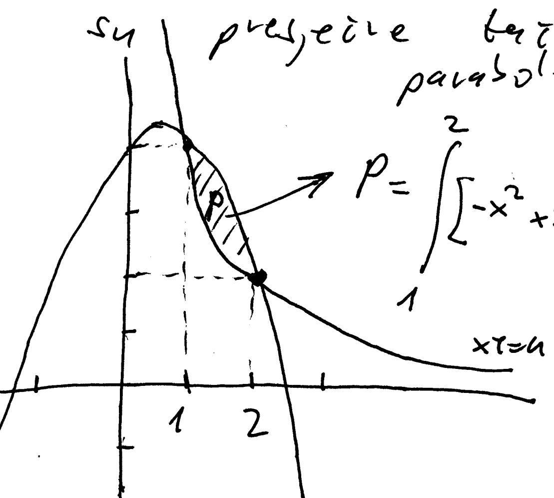
za $x = 1$: $\quad 0 = 0$ ok

$(-x^3 + x^2 + 4x - 4) : (x - 1) =$

$= -x^2 + 4$



$P_1(1,4)$, $P_2(2,2)$ i $P_3(-2,-2)$ su presječne tačke parabole i hiperbole



$$P = \int_1^2 \left[(-x^2 + x + 4) - \frac{4}{x}\right] dx$$

$$= \frac{19}{6} - 4\ln 2$$

površina

13. Naći ekstreme f-je $z = e^{x^2-y}(5-2x+y)$.

Rj. $\frac{\partial z}{\partial x} = e^{x^2-y}\cdot(2x)(5-2x+y) + e^{x^2-y}(-2) = -2e^{x^2-y}(2x^2-xy-5x+1)$

$\frac{\partial z}{\partial y} = e^{x^2-y}(-1)(5-2x+y) + e^{x^2-y}\cdot 1 = e^{x^2-y}(2x-y-4)$

$$-2e^{x^2-y}(2x^2-xy-5x+1) = 0$$
$$e^{x^2-y}(2x-y-4) = 0$$

$e^{x^2-y} > 0 \ \forall x \ i \ \forall y$

$$2x^2-xy-5x+1=0$$
$$y = 2x-4$$

$2x^2 - x(2x-4) - 5x + 1 = 0$

$\underline{2x^2} - \underline{2x^2} + 4x - 5x + 1 = 0$

$x = 1$

$y = 2-4 = -2 \qquad M(1,-2)$

$M(1,-2)$ je stacionarna tačka

$\frac{\partial^2 z}{\partial x^2} = -4e^{x^2-y}\cdot x\cdot(2x^2-xy-5x+1) + (-2)e^{x^2-y}(4x-y-5) =$

$= -2e^{x^2-y}\cdot(4x^3-2x^2y-10x^2+\underline{\underline{2x}}+\underline{\underline{4x}}-y-5) = -2e^{x^2-y}(4x^3-2x^2y-10x^2+6x-y-5)$

$\frac{\partial^2 z}{\partial x\partial y} = 2e^{x^2-y}(2x^2-xy-5x+1) - 2e^{x^2-y}(-x) = 2e^{x^2-y}(2x^2-xy-4x+1)$

$\frac{\partial^2 z}{\partial y^2} = -e^{x^2-y}(2x-y-4) + e^{x^2-y}(-1) = -e^{x^2-y}(2x-y-3)$

$M(1,-2)$,

$A = -2e^3\cdot(4+4-10+6+2-5) = -2e^3$

$B = 2e^3\cdot(2+2-4+1) = 2e^3$

$C = -e^3\,(2+2-3) = -e^3$

$D = AC - B^2$

$D = 2e^6 - 4e^6 < 0$

f-ja z u tački M nema ekstrema

14. Naći uslovne ekstreme f-je $z = x^2+xy+y^2$ ako je $4x^2+4xy+y^2 = 1$.

Rj. $F(x,y) = x^2+xy+y^2+\lambda(4x^2+4xy+y^2-1)$

$\frac{\partial F}{\partial x} = 2x+y+8\lambda x+4\lambda y = 0$

$\frac{\partial F}{\partial y} = x+2y+4\lambda x+2\lambda y = 0$

$\frac{\partial F}{\partial \lambda} = 4x^2+4xy+y^2-1 = 0$

$M_1(-\frac{1}{2},0)$, $M_2(\frac{1}{2},0)$, $\lambda = -\frac{1}{4}$

stacionarne tačke

$\frac{\partial^2 F}{\partial x^2} = 2+8\lambda$

$\frac{\partial^2 F}{\partial x\partial y} = 1+4\lambda$

$\frac{\partial^2 F}{\partial y^2} = 2+2\lambda$

$D = AC - B^2$

$D = 0$

u oba slučaja

$d^2F = \frac{3}{2}dy^2$

$z_{min} = \frac{1}{4}$

$d^2F = F''_{xx}(x_1,y_1)dx^2 + 2F''_{xy}(x_1,y_1)\,dx\,dy + F''_{yy}(x_1,y_1)\,dy^2$

15. Riješiti diferencijalnu jednačinu $(x+y-2)\,dx+(x-y+4)\,dy=0$

Rj. $(x-y+4)\,dy = -(x+y-2)\,dx$

$$y' = \frac{-x-y+2}{x-y+4}$$ ovo je difer. jedn. koja se svodi na homogenu (oblika $y' = f\left(\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}\right)$)

$a_1b_2 - a_2b_1 \neq 0$

uvodimo smjenu $x = u+\alpha$, $y = v+\beta$ gdje je

$$-\alpha-\beta+2=0$$
$$\alpha-\beta+4=0$$
$$\alpha=-1,\quad \beta=3$$

$x = u-1$, $y = v+3$ $\Rightarrow$ $u = x+1$, $v = y-3$

$y' = v'$

$$v' = \frac{-u+1-v-3+2}{u-1-v-3+4}$$

$$v' = \frac{-u-v}{u-v} \quad \begin{matrix} :u \\ :u \end{matrix}$$

$$v' = \frac{-1-\frac{v}{u}}{1-\frac{v}{u}}$$ ovo je homogena dif. jednačina (oblika $y' = f(\frac{y}{x})$).

uvodimo smjenu $z = \frac{v}{u}$

$v = u\cdot z \;/\; \frac{d}{du}$

$v' = z + z'u$

$$z+z'u = \frac{-1-z}{1-z}, \quad z' = \frac{dz}{du}$$

$$\frac{1-z}{z^2-2z-1}\,dz = \frac{1}{u}\,du \quad /\int$$

$$-\frac{1}{2}\ln|z^2-2z-1| = \ln|u| + \ln C_1$$

$$\frac{1}{\sqrt{z^2-2z-1}} = u\,C$$

$$1 = u^2 C_2 (z^2-2z-1) \quad /:C_2$$

$$u^2(z^2-2z-1) = C_3$$

$$u^2\left(\frac{v^2}{u^2} - 2\frac{v}{u} - 1\right) = C_3$$

$$y^2 - x^2 - 8y + 4x - 2xy + 14 = C_3 \quad /\cdot(-1)$$

$$x^2 - y^2 - 4x + 8y + 2xy = C$$

opšte rješenje diferencijalne jednačine

16. Riješiti diferencijalnu jednačinu

$x(2+x)y' + 2(1+x)y = 1+3x^2$, uz početni uslov $y(-1)=1$.

Rj.

$$y' + \frac{2(1+x)}{x(2+x)}y = \frac{1+3x^2}{2(1+x)}$$

ovo je linearna difer. jedn.
($y' + f(x)y = g(x)$)

uvodimo smjenu $y = uv$

$y' = u'v + uv'$

$$u'v + uv' + \frac{2(1+x)}{x(2+x)}uv = \frac{1+3x^2}{x(2+x)}$$

$$u'v + u\underbrace{\left(v' + \frac{2(1+x)}{x(2+x)}v\right)}_{=0} = \frac{1+3x^2}{x(1+x)}$$

$$\frac{dv}{dx} = -\frac{2(1+x)}{x(2+x)}v$$

$$\frac{dv}{v} = -\frac{2(1+x)}{x(2+x)}dx \quad //\int$$

$$\ln|v| = -\ln|x(2+x)|$$

$$v = \frac{1}{x(2+x)}$$

$$u' \cdot \frac{1}{x(2+x)} = \frac{1+3x^2}{x(2+x)}$$

$$u' = \frac{du}{dx}, \quad \frac{du}{dx} = 1+3x^2$$

$$u = x + x^3 + C$$

$$y = uv = \frac{x^3+x+C}{x(2+x)}$$ opšte rješenje dif. jedn.

$y(-1) = 1$

$$y = \frac{x^3+x+C}{x(2+x)}$$

$y(-1) = 1 \Rightarrow C = 1$

$$y = \frac{x^3+x+1}{x(2+x)}$$

partikularno rješenje diferencijalne jednačine