Grupa C (20.06.2011.)

1. $$D = \begin{vmatrix} a & 1 & -1 \\ 1 & a & -1 \\ 1 & -1 & -a \end{vmatrix} \overset{I_v - II_v}{=} \begin{vmatrix} a-1 & 1-a & 0 \\ 1 & a & -1 \\ 1 & -1 & -a \end{vmatrix}$$

$$= (a-1) \begin{vmatrix} 1 & -1 & 0 \\ 1 & a & -1 \\ 1 & -1 & -a \end{vmatrix} = (a-1) \begin{vmatrix} 1 & 0 & 0 \\ 1 & a+1 & -1 \\ 1 & 0 & -a \end{vmatrix}$$

$$= (a-1)\cdot(a+1)\cdot(-a) = -a(a-1)(a+1)$$

$$D_x = \begin{vmatrix} 1 & 1 & -1 \\ 1 & a & -1 \\ 1 & -1 & -a \end{vmatrix} = \begin{vmatrix} 1 & 1 & -1 \\ 0 & a-1 & 0 \\ 0 & -2 & 1-a \end{vmatrix} = (a-1)(1-a)$$
$$= -(a-1)^2$$

$$D_y = \begin{vmatrix} a & 1 & -1 \\ 1 & 1 & -1 \\ 1 & 1 & -a \end{vmatrix} = \begin{vmatrix} a & 1 & -1 \\ 1-a & 0 & 0 \\ 1-a & 0 & 1-a \end{vmatrix} = -(1-a)^2$$
$$= -(a-1)^2$$

$$D_z = \begin{vmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & -1 & 1 \end{vmatrix} = \begin{vmatrix} a & 1 & 1 \\ 1-a & a-1 & 0 \\ 1-a & -2 & 0 \end{vmatrix} = -2(1-a)-(1-a)(a-1)$$

$$= (1-a)(-2-a+1) = (1-a)(-1-a) = (a-1)(a+1)$$

$$D = 0 \iff a \in \{0, -1, 1\}$$

1° $a \neq \pm 1$, $a \neq 0$ – sistem ima tačno jedno rješenje

$$\left(\frac{a-1}{a(a+1)}, \frac{a-1}{a(a+1)}, -\frac{1}{a}\right)$$

2° $a = 0 \Rightarrow D = 0, D_x = -1$ – sistem nema rješenja

3° $a = -1 \Rightarrow D = 0, D_x = -4$ – sistem nema rješenja

4° $a=1 \Rightarrow D=D_x=D_y=D_z=0$, sistem glasi:

$$x+y-z=1$$
$$x+y-z=1 \quad \ldots(1)$$
$$x-y-z=1 \quad \ldots(2)$$

$(1)-(2): \quad 2y=0 \Rightarrow y=0 \Rightarrow x-z=1.$

$x=z+1, \; y=0, \; z$ uzimamo proizvoljno

Rješenje: $(z+1, 0, z), \; z\in R$ – beskonačno mnogo rješenja

2. $y=\ln(x^3+x^2)$

– Def. područje: $x^3+x^2>0 \Rightarrow x^2(x+1)>0$

$\Rightarrow x\neq 0$ i $x>-1$

$\Rightarrow \boxed{x\in(-1,0)\cup(0,+\infty)}$

– Nula funkcije: $x^3+x^2=1 \Rightarrow x\approx 0{,}7549=x_0$

– Znak: $x<x_0 \Rightarrow y<0$

$x>x_0 \Rightarrow y>0$

– Asimptote:

a) V.A. $\lim\limits_{x\to -1+} \ln(x^3+x^2)=\ln 0_+=-\infty$

$\lim\limits_{x\to 0-} \ln(x^3+x^2)=\ln 0_+=-\infty$

$\lim\limits_{x\to 0+} \ln(x^3+x^2)=\ln 0_+=-\infty$

$\left.\begin{array}{l} x=-1 \text{ (desna)} \\ x=0 \text{ (obostrana)} \end{array}\right\}$ vert. asimptote

b) H.A $\lim_{x\to+\infty} \ln(x^3+x^2) = \ln(+\infty) = +\infty$

Nema H.A

c) K.A. $k = \lim_{x\to+\infty} \frac{\ln(x^3+x^2)}{x} = \frac{\infty}{\infty}$

$\overset{L.H.}{=} \lim_{x\to+\infty} \frac{\frac{1}{x^3+x^2}\cdot(3x^2+2x)}{1} = \lim_{x\to+\infty} \frac{3x^2}{x^3} = 0$

Nema K.A

- Izvod: $y' = \frac{3x^2+2x}{x^3+x^2} = \frac{x(3x+2)}{x^2(x+1)} = \frac{3x+2}{x(x+1)}$

$y' = 0 \Rightarrow 3x+2 = 0 \Rightarrow x = -\frac{2}{3}$

	-1	$-\frac{2}{3}$	0	$+\infty$
$3x+2$	$-$	$+$	$+$	
x	$-$	$-$	$+$	
$x+1$	$+$	$+$	$+$	
y'	$+$	$-$	$+$	
y	$\nearrow$	$\searrow$	$\nearrow$	
	max	N.D.		

$y_{max} = y(-\frac{2}{3}) = \ln\left(-\frac{8}{27}+\frac{4}{9}\right) = \ln\frac{-8+12}{27} = \ln\frac{4}{27}$

$T_{max}\left(-\frac{2}{3}, \ln\frac{4}{27}\right)$

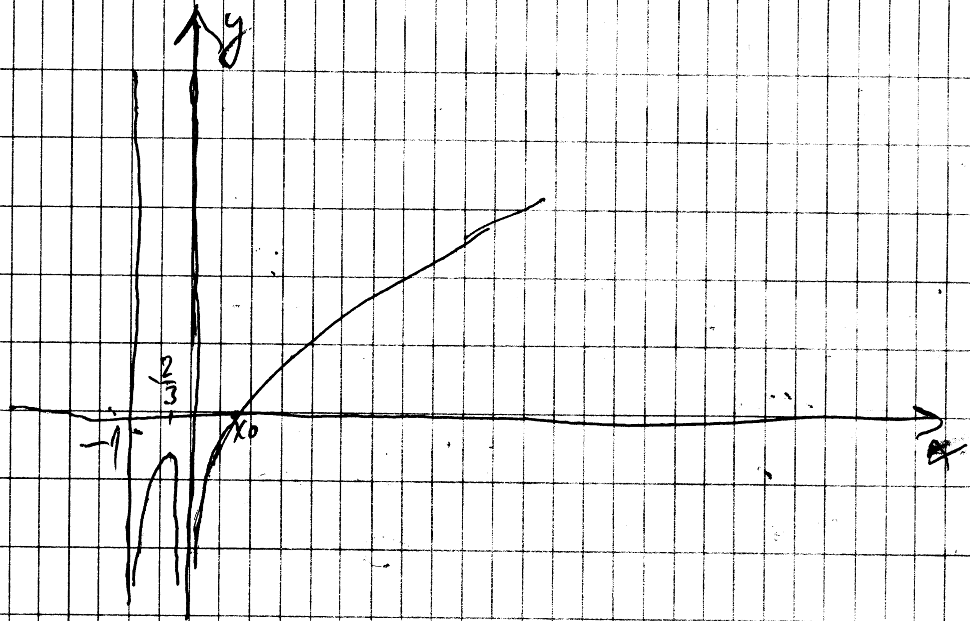
$y'' = \left(\frac{3x+2}{x^2+x}\right)'$

$= \frac{3(x^2+x)-(3x+2)(2x+1)}{(x^2+x)^2} = \frac{3x^2+\cancel{3x}-6x^2-\cancel{3x}-4x-2}{(x^2+x)^2}$

$= \frac{-3x^2-4x-2}{(x^2+x)^2} = -\frac{3x^2+4x+2}{(x^2+x)^2}$

$y''=0 \Rightarrow 3x^2+4x+2=0 \Rightarrow x_{1/2} = \frac{-4\pm\sqrt{16-24}}{6} \notin \mathbb{R}$

$y''<0$ za svako x – nema prevojnih tačaka, funkcija konkavna



3. $I = \int (x-2)\sqrt[3]{x+1}\,dx = \left| \begin{matrix} x+1 = t^3 \\ dx = 3t^2\,dt \end{matrix} \right|$

$= \int (t^3-3)\cdot t \cdot 3t^2\,dt = 3\int (t^6 - 3t^3)\,dt$

$= 3\left(\frac{t^7}{7} - 3\cdot\frac{t^4}{4}\right) + C = \frac{3t^7}{7} - 9\frac{t^4}{4} + C$

$= 3\cdot\frac{(\sqrt[3]{x+1})^7}{7} - \frac{9}{4}\cdot(\sqrt[3]{x+1})^4 + C$

4. $2x-y-1 = z \Rightarrow 2 - y' = z' \Rightarrow y' = 2-z'$

$2-z' = \sqrt{z} \Rightarrow z' = 2-\sqrt{z} \Rightarrow \frac{z'}{2-\sqrt{z}} = 1 \;/\cdot dx, \int$

$\int \frac{dz}{2-\sqrt{z}} = \int dx$ $\quad \left| \sqrt{z} = t \Rightarrow z = t^2 \Rightarrow dz = 2t\,dt \right|$

$\Rightarrow \int \frac{2t\,dt}{2-t} = \int \frac{2t-4+4}{2-t}\,dt = \int\left(-2 + \frac{4}{2-t}\right)dt$

$= -2t - 4\ln|2-t| + C = -2\sqrt{z} - 4\ln|2-\sqrt{z}| + C$

$\Rightarrow -2\sqrt{z} - 4\ln|2-\sqrt{z}| = x + C$

$-2\sqrt{2x-y-1} - 4\ln|2-\sqrt{2x-y-1}| = x + C$