Grupa B (Ispit 20.06.2011.)

1. $D = \begin{vmatrix} 0 & -m & 1 \\ -1 & -1-m+3 \\ -m & 1-m & 1 \end{vmatrix} = m\cdot(-1-m^2+3m) + \cancel{m} - 1 - \cancel{m}$

$= -m^3 + 3m^2 - m - 1$

$= (-m^3 + m^2) + (m^2 - m) + (m^2 - 1)$

$= -m^2(m-1) + m(m-1) + (m-1)(m+1)$

$= (m-1)(-m^2 + m + m + 1)$

$= (m-1)(-m^2 + 2m + 1)$

$D = 0$ ako je $m_1 = 1$ ili $-m^2 + 2m + 1 = 0$

$\Rightarrow m_{2,3} = \dfrac{-2 \pm \sqrt{4+4}}{-2} = \dfrac{-2 \pm 2\sqrt{2}}{-2} = 1 \pm \sqrt{2}$

$m = 1 \Rightarrow \vec{a} = (0,-1,1),\ \vec{b} = (-1,-1,2),\ \vec{c} = (-1,0,1)$

$\vec{c} = \alpha\vec{a} + \beta\vec{b} \Rightarrow (-1,0,1) = (0,-\alpha,\alpha) + (-\beta,-\beta,2\beta)$

$\Rightarrow -1 = -\beta,\ 0 = -\alpha - \beta,\ 1 = \alpha + 2\beta$

$\Rightarrow \beta = 1,\ \alpha = -1$

$\Rightarrow \boxed{\vec{c} = \vec{b} - \vec{a}}$

2. $y = \dfrac{x^3}{4(2-x)^2} = \dfrac{x^3}{4(x-2)^2}$

- Def. pod.: $x \in (-\infty, 2) \cup (2, +\infty)$
- Nule: $x = 0$
- Znak: $x > 0 \Rightarrow y > 0$

  $x < 0 \Rightarrow y < 0$
- Vertikalna asimptota: $x = 2$,

$\lim\limits_{x\to 2-} \dfrac{x^3}{4(x-2)^2} = \dfrac{8}{0_+} = +\infty$

$\lim\limits_{x\to 2+} \dfrac{x^3}{4(x-2)^2} = \dfrac{8}{0_+} = +\infty$

- Horizontalne asimptote nema, jer

$$\lim_{x\to\pm\infty} \frac{x^3}{4(x-2)^2} = \lim_{x\to\pm\infty} \frac{x^3}{4x^2} = \lim_{x\to\pm\infty} \frac{x}{4} = \pm\infty.$$

- Kosa asimptota: $y = \frac{1}{4}x + 1$

$$k = \lim_{x\to\pm\infty} \frac{\frac{x^3}{4(x-2)^2}}{\frac{x}{1}} = \lim_{x\to\pm\infty} \frac{x^{3^2}}{4x(x-2)^2} =$$

$$= \lim_{x\to\pm\infty} \frac{x^2}{4x^2} = \frac{1}{4}.$$

$$n = \lim_{x\to\pm\infty} \left(\frac{x^3}{4(x-2)^2} - \frac{1}{4}x\right) = \lim_{x\to\pm\infty} \frac{x^3 - x(x-2)^2}{4(x-2)^2}$$

$$= \lim_{x\to\pm\infty} \frac{x^3 - x(x^2-4x+4)}{4(x-2)^2} =$$

$$= \lim_{x\to\pm\infty} \frac{\cancel{x^3} - \cancel{x^3} + 4x^2 - 4x}{4(x^2-4x+4)} = 1$$

x	0	4
y	1	2

- Izvod: $y' = \frac{3x^2\cdot 4(x-2)^2 - x^3\cdot 4\cdot 2(x-2)}{16(x-2)^4}$

$$= \frac{4x^2(x-2)\left[3(x-2) - 2x\right]}{16(x-2)^4}$$

$$= \frac{4x^2(x-2)(3x-6-2x)}{16(x-2)^4} = \frac{x^2(x-6)}{4(x-2)^3}$$

$y' = 0 \Rightarrow x_1 = 0,\ x_2 = 6$

	$-\infty$ … 2	2 … 6	6 … $+\infty$
$x-6$	$-$	$-$	$+$
$(x-2)^3$	$-$	$+$	$+$
y'	$+$	$-$	$+$
y	$\nearrow$	$\searrow$	$\nearrow$
		N.D.	min

$$y(6) = \frac{6^3}{4\cdot 4^2} = \frac{2^3\cdot 3^3}{(2^2)^3} = \frac{3^3}{2^3} = \frac{27}{8}$$

$T_{min}\left(6, \frac{27}{8}\right)$

$$y'' = \left[\frac{x^3-6x^2}{4(x-2)^3}\right]' = \frac{1}{4}\cdot\frac{(3x^2-12x)(x-2)^3-(x^3-6x^2)\cdot 3(x-2)^2}{(x-2)^6}$$

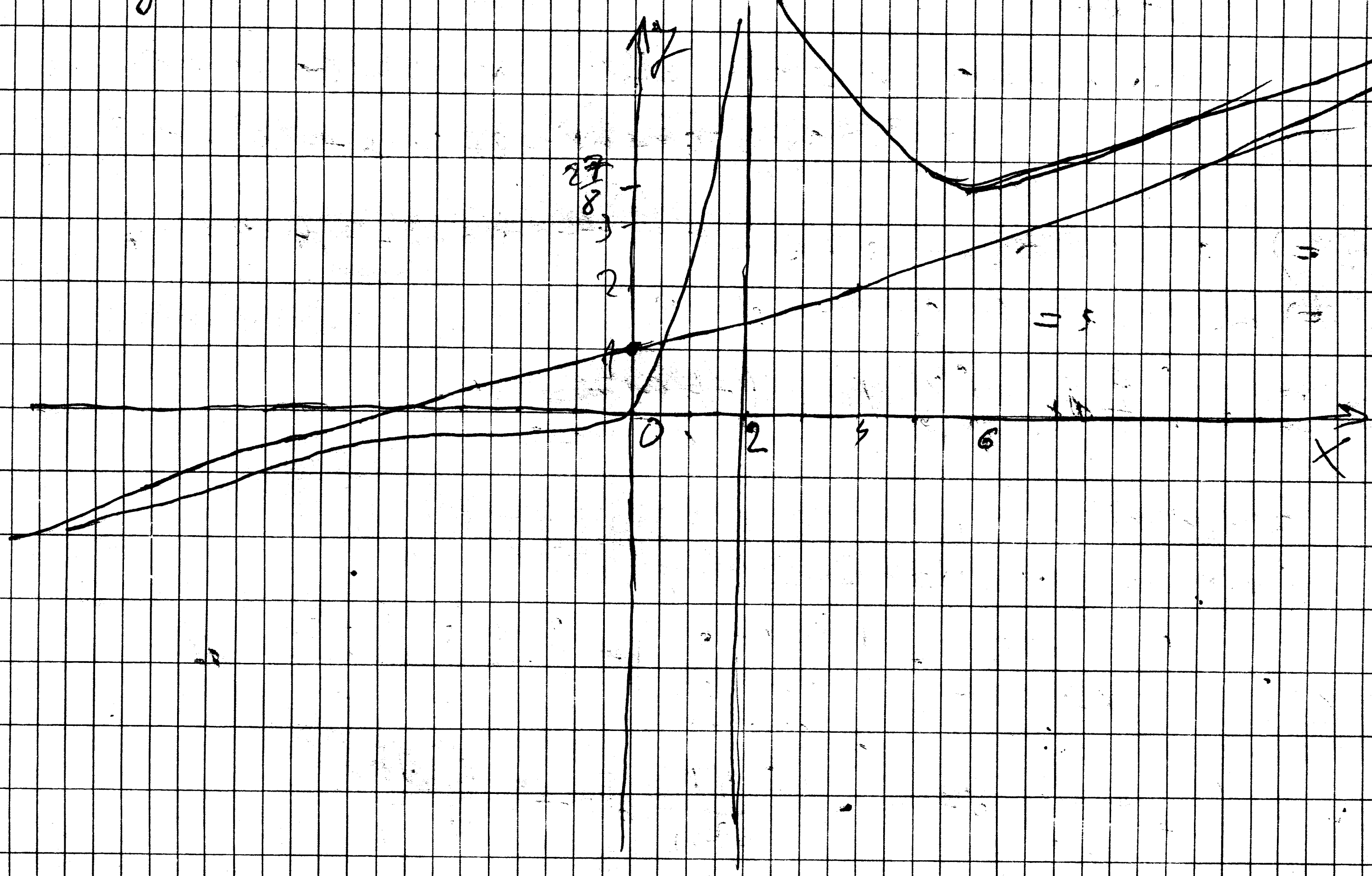
$$= \frac{1}{4}\cdot\frac{3\cancel{(x-2)^2}\left[(x^2-4x)(x-2)-(x^3-6x^2)\right]}{(x-2)^{\cancel{6}\,4}}$$

$$= \frac{1}{4}\cdot\frac{3\cdot(\cancel{x^3}-\cancel{2x^2}-\cancel{4x^2}+8x-\cancel{x^3}+\cancel{6x^2})}{(x-2)^4}$$

$$= \frac{6x}{(x-2)^4}$$

$y''=0 \Rightarrow x=0$

$$\left.\begin{array}{l} y''>0 \text{ za } x>0 \\ y''<0 \text{ za } x<0 \end{array}\right\} \Rightarrow \text{prevojna tačka } P(0,0)$$



3. $I = \int \frac{x^5 - 2x^3 + 4x + 4}{x^4 - 2x^3 + 2x^2}\, dx$

$(x^5 - 2x^3 + 4x + 4) : (x^4 - 2x^3 + 2x^2) = x + 2 + \frac{-4x^2 + 4x + 4}{x^4 - 2x^3 + 2x^2}$

$\underline{-x^5 \mp 2x^4 \pm 2x^3}$

$= 2x^4 - 4x^3 + 4x + 4$

$\underline{-2x^4 \mp 4x^3 \pm 4x^2}$

$= \quad = -4x^2 + 4x + 4$

$I = \int \left(x + 2 + \frac{-4x^2 + 4x + 4}{x^4 - 2x^3 + 2x^2} \right) dx$

$= \frac{x^2}{2} + 2x + I_1, \quad I_1 = \int \frac{-4x^2 + 4x + 4}{x^4 - 2x^3 + 2x^2}\, dx$

$\frac{-4x^2 + 4x + 4}{x^2(x^2 - 2x + 2)} = \frac{a}{x} + \frac{b}{x^2} + \frac{cx + d}{x^2 - 2x + 2} \Big/ \cdot x^2(x^2 - 2x + 2)$

$-4x^2 + 4x + 4 = ax(x^2 - 2x + 2) + b(x^2 - 2x + 2) + (cx + d) \cdot x^2$

$= ax^3 - 2ax^2 + 2ax + bx^2 - 2bx + 2b + cx^3 + dx^2$

$\Rightarrow a + c = 0, \; -2a + b + d = -4, \; 2a - 2b = 4, \; 2b = 4$

$\Rightarrow b = 2, \; a = 4, \; c = -4, \; d = 2$

$I_1 = \int \frac{4}{x}\, dx + \int \frac{2}{x^2}\, dx + \int \frac{-4x + 2}{x^2 - 2x + 2}\, dx$

$= 4\ln|x| - \frac{2}{x} + I_2$

$x^2 - 2x + 2 = x^2 - 2x + 1 + 1 = (x-1)^2 + 1$

$I_2 = \int \frac{-4x + 2}{(x-1)^2 + 1} = \left| \begin{array}{l} x - 1 = t \\ dx = dt \\ x = t + 1 \end{array} \right| = \int \frac{-4(t+1) + 2}{t^2 + 1}\, dt$

$= \int \frac{-4t - 2}{t^2 + 1}\, dt = -2 \int \frac{2t}{t^2 + 1}\, dt - 2 \int \frac{dt}{t^2 + 1} =$

$= -2\ln(t^2 + 1) - 2\,\mathrm{arctg}\, t + C$

nastavak 3.

$= -2\ln((x-1)^2+1) - 2\operatorname{arctg}(x-1) + C$

$I = \frac{x^2}{2} + 2x + 4\ln|x| - \frac{2}{x} - 2\ln(x^2-2x+2) - 2\operatorname{arctg}(x-1) + C$

4.
$$y' = \frac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2} \quad \Big/ : x^2$$

$$y' = \frac{\frac{y^2}{x^2} - 2\cdot\frac{y}{x} - 1}{\frac{y^2}{x^2} + 2\cdot\frac{y}{x} - 1}$$

Smjena: $\frac{y}{x} = z = z(x) \Rightarrow y = xz \Rightarrow y' = z + xz'$

$$z + xz' = \frac{z^2 - 2z - 1}{z^2 + 2z - 1}$$

$$xz' = \frac{z^2 - 2z - 1}{z^2 + 2z - 1} - z$$

$$xz' = \frac{z^2 - 2z - 1 - z(z^2 + 2z - 1)}{z^2 + 2z - 1}$$

$$xz' = \frac{z^2 - 2z - 1 - z^3 - 2z^2 + z}{z^2 + 2z - 1}$$

$$xz' = \frac{-z^3 - z^2 - z - 1}{z^2 + 2z - 1} \quad \Big/ \cdot \frac{z^2 + 2z - 1}{z^3 + z^2 + z + 1} \cdot \frac{1}{x}$$

$$z' \cdot \frac{z^2 + 2z - 1}{z^3 + z^2 + z + 1} = -\frac{1}{x} \quad \Big/ \cdot dx, \int$$

$$z^3 + z^2 + z + 1 = z^2(z+1) + (z+1) = (z+1)(z^2+1)$$

$$\int \frac{z^2 + 2z - 1}{(z+1)(z^2+1)}\, dz = -\int \frac{1}{x}\, dx$$

$$\frac{z^2+2z-1}{(z+1)(z^2+1)} = \frac{a}{z+1} + \frac{bz+c}{z^2+1} \quad \Big/ \cdot (z+1)(z^2+1)$$

$$z^2+2z-1 = a(z^2+1) + (bz+c)(z+1)$$

$$= az^2 + a + bz^2 + bz + cz + c$$

$$\Rightarrow a+b=1, \quad b+c=2, \quad a+c=-1.$$

$$\Rightarrow a=-1, \; b=2, \; c=0$$

$$\Rightarrow -\ln|z+1| + \ln(z^2+1) = -\ln|x| + \ln C$$

$$\ln\frac{z^2+1}{|z+1|} = \ln\frac{C}{|x|}$$

$$\Rightarrow \frac{z^2+1}{z+1} = \frac{C}{x} \Rightarrow C(z+1) = x(z^2+1)$$

$$\Rightarrow C\left(\frac{y}{x}+1\right) = x\left(\frac{y^2}{x^2}+1\right)$$

$$C\cdot\frac{y}{x} + C = \frac{y^2}{x} + x \quad \Big/ \cdot x$$

$$Cy + Cx = y^2 + x^2$$