Grupa C EF ispit 5. juli 2011.

1. $A X^{-1} B = (B^T)^{-1} \cdot A \quad / \cdot A^{-1}$ lijevo, B^{-1} desno

$$X^{-1} = A^{-1} \cdot (B^T)^{-1} \cdot A \cdot B^{-1} \quad / \text{ inverz}$$

$$X = B \cdot A^{-1} \cdot B^T \cdot A$$

$$\det(A) = \begin{vmatrix} 1 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0 - 2 = -2 \Rightarrow A \text{ je regularna}$$

$A_{11} = 0$, $A_{21} = -2$, $A_{31} = 0$

$A_{12} = -1$, $A_{22} = 1$, $A_{32} = 0$

$A_{13} = 0$, $A_{23} = 0$, $A_{33} = -2$

$$\Rightarrow A^* = \begin{bmatrix} 0 & -2 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-2} \cdot A^* = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det(B) = \begin{vmatrix} 1 & 0 & 2 \\ 1 & -2 & 1 \\ 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 2 \\ 0 & -2 & -1 \\ 0 & 1 & 1 \end{vmatrix} = -2 + 1 = -1 \Rightarrow B \text{ je regularna}$$

$$X = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 2 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2 \\ -1 & 2 & 1 \\ \frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 & 0 \\ -2 & 0 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 & 3 \\ -3 & 2 & 3 \\ 5 & 5 & \frac{1}{2} \end{bmatrix}$$

2. $y=\dfrac{4x^2+ax+b}{x+1}$, $x_{1,2}=-\frac{1}{2}\pm i\sqrt{2}$

$x_1+x_2=-1=-\frac{a}{4} \Rightarrow a=4$

$x_1\cdot x_2=\left(-\frac{1}{2}\right)^2-(i\sqrt{2})^2=\frac{1}{4}-2i^2=\frac{1}{4}+2=\frac{9}{4}$.

$\frac{9}{4}=\frac{b}{4} \Rightarrow b=9$

$$y=\frac{4x^2+4x+9}{x+1}$$

Def. područje: $x\neq -1 \Rightarrow x\in(-\infty,-1)\cup(-1,+\infty)$.

Nule: nema realnih nula

Znak: $x>-1 \Rightarrow y>0$

$x<-1 \Rightarrow y<0$

asimptote:

V.A. $x=-1$, $\lim\limits_{x\to -1^-}\dfrac{4x^2+4x+9}{x+1}=\dfrac{9}{0_-}=-\infty$

$\lim\limits_{x\to -1^+}\dfrac{4x^2+4x+9}{x+1}=\dfrac{9}{0+}=+\infty$

$\lim\limits_{x\to\pm\infty}\dfrac{4x^2+4x+9}{x+1}=\lim\limits_{x\to\pm\infty}\dfrac{4x^{\cancel{2}}}{\cancel{x}}=\pm\infty \Rightarrow$ nema H.A.

kosa: $y=kx+n$

$k=\lim\limits_{x\to\pm\infty}\dfrac{\frac{4x^2+4x+9}{x+1}}{x}=\lim\limits_{x\to\pm\infty}\dfrac{4x^2}{x^2}=4$

$n=\lim\limits_{x\to\pm\infty}\left(\dfrac{4x^2+4x+9}{x+1}-4x\right)=\lim\limits_{x\to\pm\infty}\dfrac{\cancel{4x^2}+\cancel{4x}+9-\cancel{4x^2}-\cancel{4x}}{x+1}=0$

$y=4x$ - kosa asimptota

x	0	1
y	0	4

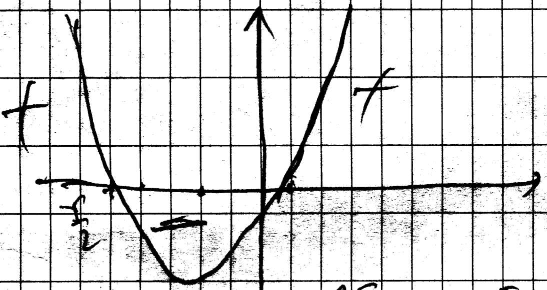
$$y' = \left(\frac{4x^2+4x+9}{x+1}\right)' = \frac{(8x+4)(x+1)-(4x^2+4x+9)}{(x+1)^2}$$

$$= \frac{8x^2+8x+4x+4-4x^2-4x-9}{(x+1)^2}$$

$$= \frac{4x^2+8x-5}{(x+1)^2}$$

$$y'=0 \Rightarrow 4x^2+8x-5=0 \Rightarrow x_{1,2} = \frac{-8\pm\sqrt{64+80}}{8} = \frac{-8\pm 12}{8}$$

$$x_1 = \frac{-20}{8} = -\frac{5}{2}\ ; \quad x_2 = \frac{4}{8} = \frac{1}{2}$$



	$-\infty$	$-\frac{5}{2}$		$\frac{1}{2}$	$+\infty$
y'	+	•	−	•	+
y	↗		↘		↗
		max		min	

$$y\left(-\frac{5}{2}\right) = \frac{4\cdot\frac{25}{4}+4\left(-\frac{5}{2}\right)+9}{-\frac{5}{2}+1} = \frac{25-10+9}{-\frac{3}{2}} = -16$$

$$y\left(\frac{1}{2}\right) = \frac{4\cdot\frac{1}{4}+4\cdot\frac{1}{2}+9}{\frac{1}{2}+1} = \frac{1+2+9}{\frac{3}{2}} = 8$$

$T_{max}\left(-\frac{5}{2}, -16\right)$, $T_{min}\left(\frac{1}{2}, 8\right)$

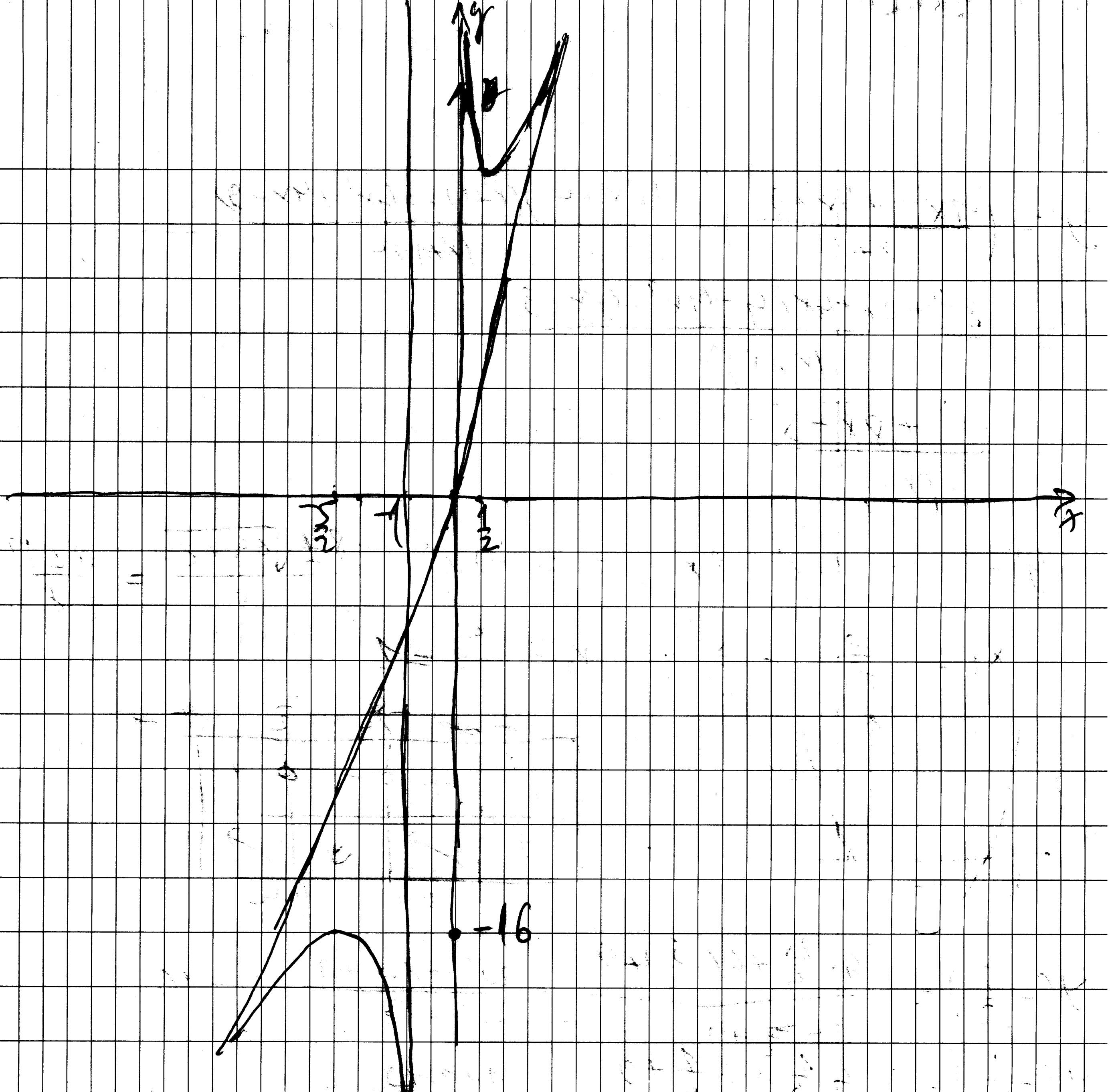
$$y'' = \left(\frac{4x^2+8x-5}{(x+1)^2}\right)' = \frac{(8x+8)(x+1)^2-(4x^2+8x-5)\cdot 2(x+1)}{(x+1)^4}$$

$$= \frac{\cancel{(x+1)}\cdot\left[(8x+8)(x+1)-2(4x^2+8x-5)\right]}{(x+1)^{\cancel{4}\,3}}$$

$$= \frac{\cancel{8x^2}+\cancel{8x}+\cancel{8x}+8-\cancel{8x^2}-\cancel{16x}+10}{(x+1)^3} = \frac{18}{(x+1)^3} \neq 0$$

Nema prevojnih tačaka;

$x > -1 \Rightarrow$ funkcija konveksna; $x < -1 \Rightarrow$ funkcija konkavna



y
x
$-\frac{3}{2}$
-1
$\frac{1}{2}$
-16

3. $y^2 = 5x - 4$

$y = 0 \Rightarrow 5x = 4 \Rightarrow x = \frac{4}{5}$

$x = 1 \Rightarrow y = 1$

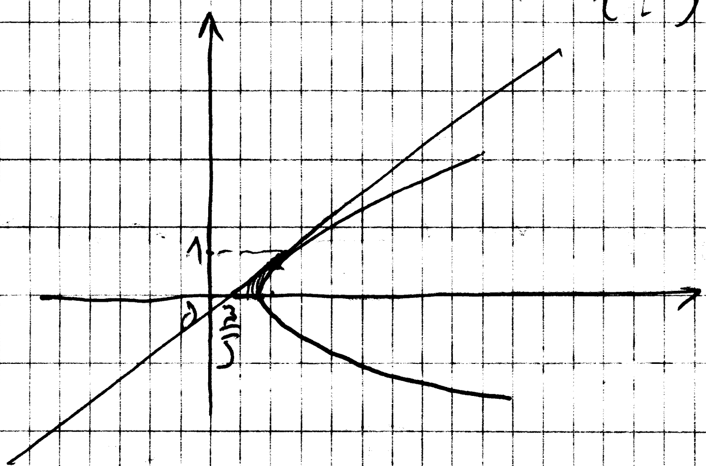
$2y \cdot y' = 5 \Rightarrow y' = \frac{5}{2y} \Rightarrow k_t = \frac{5}{2}$

$t: y - 1 = \frac{5}{2}(x-1)$

$y = \frac{5}{2}x - \frac{5}{2} + 1$

$y = \frac{5}{2}x - \frac{3}{2}$

$\frac{5x-3}{2} = 0 \Rightarrow x = \frac{3}{5}$



$y = \frac{5x-3}{2} \Rightarrow 5x - 3 = 2y \Rightarrow 5x = 2y + 3 \Rightarrow x = \frac{2y+3}{5}$

$y^2 = 5x - 4 \Rightarrow y^2 + 4 = 5x \Rightarrow x = \frac{y^2+4}{5}$

$P = \int_0^1 \left(\frac{y^2+4}{5} - \frac{2y+3}{5} \right) dy = \int_0^1 \frac{y^2 - 2y + 1}{5}\, dy =$

$= \frac{1}{5} \int_0^1 (y-1)^2 dy = \left| \begin{matrix} y - 1 = t \\ dy = dt \end{matrix} \right| = \frac{1}{5} \int_{-1}^{0} t^2 dt$

$= \frac{1}{5} \cdot \frac{t^3}{3} \Big|_{-1}^{0} = \frac{1}{15}(0 + 1) = \frac{1}{15}$

4. $x \cdot (y')^2 - yy' + \frac{a}{2} = 0$

$x \cdot (y')^2 + \frac{a}{2} = yy' \quad / : y'$

$x \cdot y' + \frac{a}{2y'} = y$ – Klerova d.j.

$y' = p \Rightarrow y = xp + \frac{a}{2p} \quad / d$

$dy = p\,dx + x\,dp - \frac{a}{2p^2}\,dp$

$p\,dx - p\,dx = dp\left(x - \frac{a}{2p^2}\right)$

$dp\left(x - \frac{a}{2p^2}\right) = 0$

$dp = 0 \Rightarrow p = c \Rightarrow y = xc + \frac{a}{2c}$ – opće rješenje

$x - \frac{a}{2p^2} = 0 \Rightarrow x = \frac{a}{2p^2} \Rightarrow 2p^2 x = a$

$p^2 = \frac{a}{2x} \Rightarrow p = \pm\sqrt{\frac{a}{2x}} \quad (x > 0)$

$y = x \cdot \left(\pm\sqrt{\frac{a}{2x}}\right) + \frac{a}{2\cdot\left(\pm\sqrt{\frac{a}{2x}}\right)}$

$y = \pm\sqrt{\frac{a}{2x}\cdot x^2} \pm \sqrt{\frac{a^2}{4\cdot\frac{a}{2x}}}$

$y = \pm\sqrt{\frac{ax}{2}} \pm \sqrt{\frac{ax}{2}} = 2\cdot\left(\pm\sqrt{\frac{ax}{2}}\right)$

$y = \pm\sqrt{\frac{ax}{2}\cdot 4} = \pm\sqrt{2ax}$

$y = \pm\sqrt{2ax}$ – singularna rješenja jednadžbe

$x = -\frac{a}{18} \Rightarrow y = 0$ uvrstimo u opće rješenje

$0 = -\frac{a}{18}\cdot c + \frac{a}{2c} \quad / \cdot 18c$

EKONOMSKI FAKULTET
U ZENICI
ZENICA, Travnička cesta 1

$$0 = -ac^2 + 9a$$

$$0 = a(9 - c^2)$$

$$c^2 = 9 \Rightarrow c = \pm 3$$

Imamo 2 partikularna rješenja:

$$y = 3x + \frac{9}{6}$$ i

$$y = -3x - \frac{9}{6}$$