# Grupa A (05.07.2011.)

1.

$$\sqrt[8]{16} = \sqrt[8]{16(\cos 0 + i\sin 0)} =$$

$$= \sqrt[8]{16}\cdot\left(\cos\frac{2k\pi}{8} + i\sin\frac{2k\pi}{8}\right),\ k = 0,1,\dots,7$$

$$\sqrt[8]{16} = \sqrt[8]{2^4} = \sqrt{2}$$

$$z_k = \sqrt{2}\cdot\left(\cos\frac{k\pi}{4} + i\sin\frac{k\pi}{4}\right),\ k = 0,1,\dots,7$$

$$z_0 = \sqrt{2}\cdot(\cos 0 + i\sin 0) = \sqrt{2}$$

$$z_1 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) = \sqrt{2}\cdot\left(\frac{\sqrt{2}}{2} + i\cdot\frac{\sqrt{2}}{2}\right) = 1+i$$

$$z_2 = \sqrt{2}\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right) = \sqrt{2}\,i$$

$$z_3 = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\,\frac{\sqrt{2}}{2}\right) = -1+i$$

$$z_4 = \sqrt{2}(\cos\pi + i\sin\pi) = -\sqrt{2}$$

$$z_5 = \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} - i\,\frac{\sqrt{2}}{2}\right) = -1-i$$

$$z_6 = \sqrt{2}\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = -\sqrt{2}\,i$$

$$z_7 = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} - i\,\frac{\sqrt{2}}{2}\right) = 1-i$$

$$\sqrt[8]{16} \in \left\{\pm\sqrt{2},\ \pm\sqrt{2}\,i,\ \pm(1+i),\ \pm(1-i)\right\}$$

2. $y = \frac{x^3(-3x+4)}{(x-1)^3}$

- Def. područje: $x \neq 1 \Rightarrow x \in (-\infty, 1) \cup (1, +\infty)$
- funkcija je ni parna ni neparna
- Nule: $x_1 = 0$, $-3x+4 = 0 \Rightarrow x_2 = \frac{4}{3}$

- Znak:

| | $-\infty$ … 0 | 0 … 1 | 1 … $\frac{4}{3}$ | $\frac{4}{3}$ … $+\infty$ |
|---|---|---|---|---|
| $x^3$ | $-$ | $+$ | $+$ | $+$ |
| $-3x+4$ | $+$ | $+$ | $+$ | $-$ |
| $(x-1)^3$ | $-$ | $-$ | $+$ | $+$ |
| $y$ | $+$ | $-$ | $+$ | $-$ |

- vertikalna asimptota:

$$\left.\begin{aligned} &\lim_{x\to 1-} \frac{x^3(-3x+4)}{(x-1)^3} = \frac{1}{0-} = -\infty \\ &\lim_{x\to 1+} \frac{x^3(-3x+4)}{(x-1)^3} = \frac{1}{0+} = +\infty \end{aligned}\right\} \text{VA } x = 1$$

- horizontalna asimptota

$$\lim_{x\to\pm\infty} \frac{x^3(-3x+4)}{(x-1)^3} = \lim_{x\to\pm\infty} \frac{\cancel{x^3}\cdot(-3x)}{\cancel{x^3}} = \mp\infty \text{ nema H.A}$$

- kosa asimptota $y = kx + n$

$$k = \lim_{x\to\pm\infty} \frac{\cancel{x^3}^{2}(-3x+4)}{\cancel{x}(x-1)^3} = \lim_{x\to\pm\infty} \frac{x^2(-3x)}{x^3} = -3$$

$$n = \lim_{x\to\pm\infty} \left( \frac{x^3(-3x+4)}{(x-1)^3} + 3x \right) =$$

$$= \lim_{x\to\pm\infty} \frac{-3x^4 + 4x^3 + 3x(x^3 - 3x^2 + 3x - 1)}{(x-1)^3}$$

$$= \lim_{x\to\pm\infty} \frac{\cancel{-3x^4} + 4x^3 + \cancel{3x^4} - 9x^3 + 9x^2 - 3x}{(x-1)^3} =$$

$$= \lim_{x\to\pm\infty} \frac{-5x^3}{x^3} = -5$$

K.A. $y = -3x - 5$

| $x$ | $-2$ | $-1$ | $0$ |
|---|---|---|---|
| $y$ | $1$ | $-2$ | $-5$ |

Izvodi:

$$y' = \left(\frac{-3x^4 + 4x^3}{(x-1)^3}\right)'$$

$$= \frac{(-12x^3 + 12x^2)(x-1)^3 - (-3x^4 + 4x^3)\cdot 3(x-1)^2}{(x-1)^6}$$

$$= \frac{12x^2(1-x)(x-1)^3 - 3(x-1)^2 \cdot x^3(4-3x)}{(x-1)^6}$$

$$= \frac{3x^2\cancel{(x-1)^2}\left[4(1-x)(x-1) - x(4-3x)\right]}{(x-1)^{\cancel{6}\,4}}$$

$$= \frac{3x^2(\cancel{4x} - 4 - 4x^2 + 4x - \cancel{4x} + 3x^2)}{(x-1)^5}$$

$$= \frac{3x^2(-x^2 + 4x - 4)}{(x-1)^5} = -\frac{3x^2(x-2)^2}{(x-1)^5} \leq 0$$

Funkcija nema ekstrema, monotono je opadajuća

$$y'' = -3\cdot\frac{\left[2x(x-2)^2 + x^2\cdot 2(x-2)\right](x-1)^4 - x^2(x-2)^2\cdot 4(x-1)^3}{(x-1)^8}$$

$$= -3\cdot\frac{2x(x-2)(x-2+x)(x-1)^4 - x^2(x-2)^2\cdot 4\cancel{(x-1)}^3}{(x-1)^8}$$

$$= -3\cdot\frac{2x(x-2)\cancel{(x-1)^3}\left[(2x-2)(x-1) - 2x(x-2)\right]}{(x-1)^{\cancel{8}\,5}}$$

$$= -3\cdot\frac{2x(x-2)(\cancel{2x^2} - 2x - 2x + 2 - \cancel{2x^2} + 4x)}{(x-1)^5}$$

$$= \frac{-12x(x-2)}{(x-1)^5}$$

$y'' = 0 \Rightarrow x_1 = 0, \quad x_2 = 2$

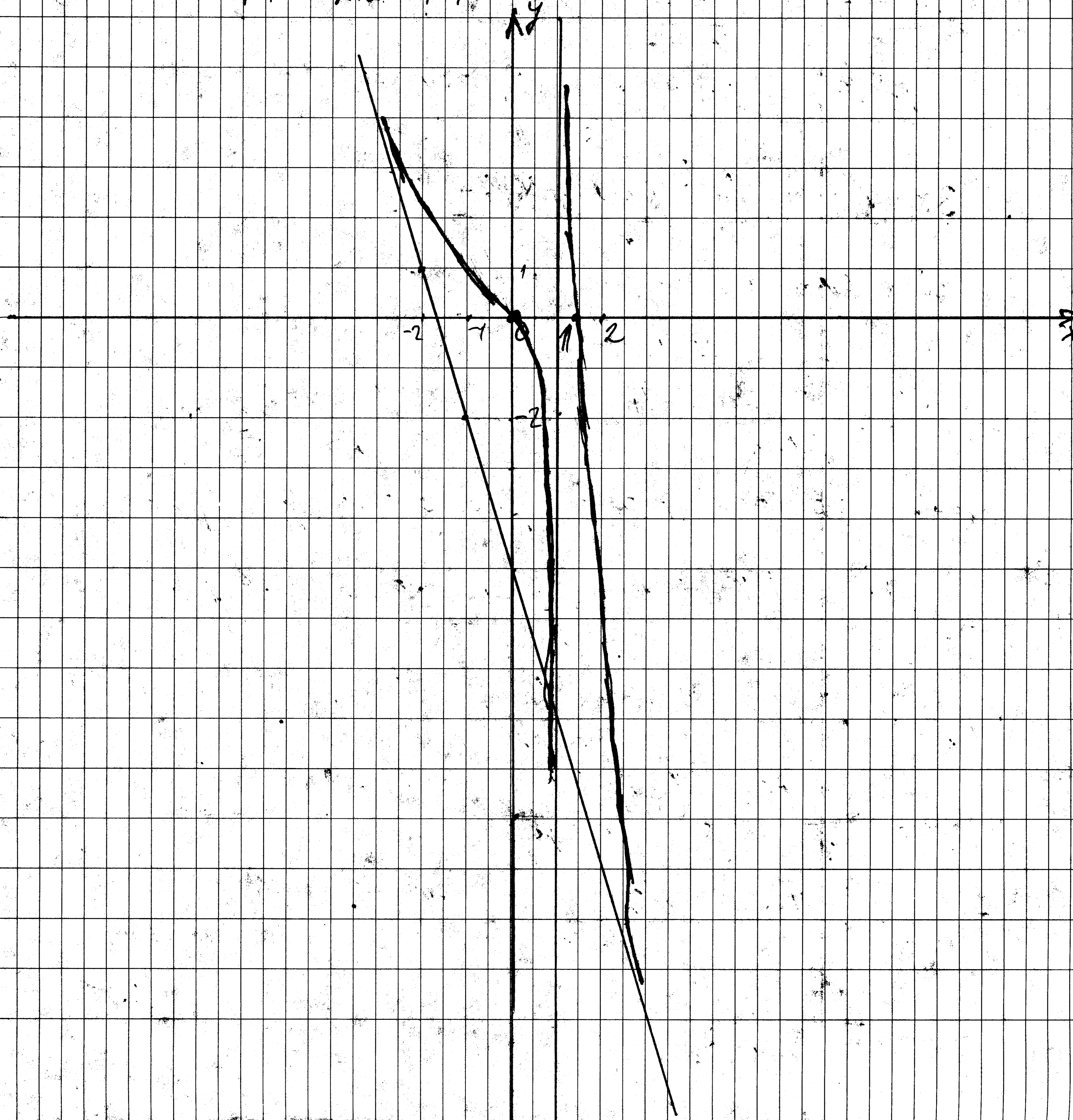
| | $-\infty$ … 0 | 0 … 1 | 1 … 2 | 2 … $+\infty$ |
|---|---|---|---|---|
| $-12x$ | + 0 | − | − | − |
| $x-2$ | − | − | − 0 | + |
| $(x-1)^5$ | − | − 0 | + | + |
| $y''$ | + | − | + | − |
| $y$ | ∪ | ∩ | ∪ | ∩ |
| | P.T. | N.O. | P.T. | |

$y(0) = 0$

$y(2) = \dfrac{8 \cdot (-2)}{1} = -16$

$P_1(0,0)$, $P_2(2,-16)$

– prevojne tačke



3) $2x+3\sqrt{3}y-12=0 \Rightarrow 2x=12-3\sqrt{3}y \Rightarrow x=6-\frac{3\sqrt{3}}{2}y$

$4x^2+9y^2=36$

$4\cdot\left(6-\frac{3\sqrt{3}}{2}y\right)^2+9y^2=36$

$4\left(36-2\cdot 6\cdot\frac{3\sqrt{3}}{2}y+\frac{9\cdot 3y^2}{4}\right)+9y^2=36$

$144-72\sqrt{3}y+27y^2+9y^2=36$

$36y^2-72\sqrt{3}y+108=0 \quad /:36$

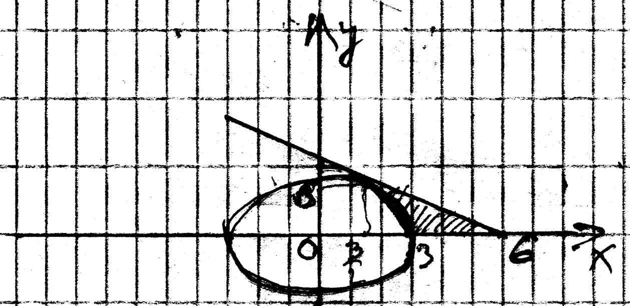
$y^2-2\sqrt{3}y+3=0$

$y^2-2\sqrt{3}y+(\sqrt{3})^2=0 \Rightarrow (y-\sqrt{3})^2=0$

$y_1=y_2=\sqrt{3} \Rightarrow x_1=x_2=6-\frac{3\sqrt{3}}{2}\cdot\sqrt{3}$

$x=6-\frac{9}{2}=\frac{3}{2}$

Prava dodiruje elipsu u tački $T\left(\frac{3}{2},\sqrt{3}\right)$




$4x^2+9y^2=36 \quad /:36$

$\frac{x^2}{9}+\frac{y^2}{4}=1$

$a=3, b=2$

$2x+3\sqrt{3}y=12 \quad /:12$

$\frac{x}{6}+\frac{y}{\frac{12}{3\sqrt{3}}}=1$

$\frac{x}{6}+\frac{y}{\frac{4}{\sqrt{3}}}=1$

$4x^2+9y^2=36$

$x^2=\frac{36-9y^2}{4}=9-\left(\frac{3y}{2}\right)^2$

$P=\int_0^{\sqrt{3}}\left[6-\frac{3\sqrt{3}}{2}y-\sqrt{9-\left(\frac{3y}{2}\right)^2}\right]dy$

$$P=\int_0^{\sqrt3} 6\,dy - \int_0^{\sqrt3}\frac{3\sqrt3}{2}\,y\,dy - \int_0^{\sqrt3}\sqrt{9\left(1-\frac{y^2}{4}\right)}\,dy$$

$$= 6\cdot y\Big|_0^{\sqrt3} - \frac{3\sqrt3}{2}\cdot\frac{y^2}{2}\Big|_0^{\sqrt3} - 3\underbrace{\int_0^{\sqrt3}\sqrt{1-\left(\frac{y}{2}\right)^2}\,dy}_{I}$$

$$= 6\cdot\sqrt3 - \frac{3\sqrt3}{4}\cdot 3 - 3I$$

$$= \frac{24\sqrt3-9\sqrt3}{4} - 3I = \frac{15\sqrt3}{4} - \boxed{I}$$

$$I = \left|\begin{array}{ll} \frac{y}{2}=\sin t & y=0 \Rightarrow t=0 \\ dy = 2\cos t\,dt & y=\sqrt3 \Rightarrow t=\frac{\pi}{3} \end{array}\right|$$

$$= \int_0^{\frac{\pi}{3}}\sqrt{1-\sin^2 t}\;2\cos t\,dt = 2\int_0^{\frac{\pi}{3}}\cos^2 t\,dt =$$

$$= \int_0^{\frac{\pi}{3}}(1+\cos 2t)\,dt = \int_0^{\frac{\pi}{3}}dt + \int_0^{\frac{\pi}{3}}\cos 2t\,dt =$$

$$= \frac{\pi}{3} + \frac12\sin 2t\Big|_0^{\frac{\pi}{3}} = \frac{\pi}{3} + \frac12\cdot\frac{\sqrt3}{2} = \frac{\pi}{3} + \frac{\sqrt3}{4}$$

$$P = \frac{15\sqrt3}{4} - \frac{\pi}{3} - \frac{\sqrt3}{4} = \cancel{\frac{14\sqrt3}{4} - \frac{\pi}{3}} = \frac{7\sqrt3}{2} - \frac{\pi}{3}$$

4. $y = 2xy' + 4(y')^3$ – Lagranžova dj.

$y' = p \Rightarrow y = 2xp + 4p^3 \quad / d$

$dy = 2(x\,dp + p\,dx) + 12p^2\,dp$

$p\,dx - 2p\,dx = 2x\,dp + 12p^2\,dp$

$-p\,dx = (2x + 12p^2)\,dp$

$p \neq 0 \Rightarrow \dfrac{dx}{dp} = \dfrac{2x + 12p^2}{-p}$

$x' + \dfrac{2x}{p} = -12p$

$x = uv \Rightarrow x' = u'v + uv'$

$u'v + uv' + \dfrac{2uv}{p} = -12p$

$u'v + u\left(v' + \dfrac{2v}{p}\right) = -12p$

$v' + \dfrac{2v}{p} = 0 \Rightarrow \dfrac{v'}{v} = -\dfrac{2}{p} \Rightarrow \dfrac{dv}{v} = -\dfrac{2}{p}\,dp$

$\ln v = -2\ln p \Rightarrow \boxed{v = p^{-2}}$

$u'v = -12p$

$u' \cdot p^{-2} = -12p \Rightarrow u' = -12p^3$

$u = -12\int p^3\,dp = -12 \cdot \dfrac{p^4}{4} + C$

$u = -3p^4 + C$

$$x = (-3p^2 + c) \cdot \frac{1}{p^2}$$

$$x = -3p^2 + \frac{c}{p^2}$$

Opće rješenje:
$$\begin{cases} x = -3p^2 + \frac{c}{p^2} \\ y = 2p\left(-3p^2 + \frac{c}{p^2}\right) + 4p^3 \end{cases}$$

$$\Rightarrow \begin{cases} x = -3p^2 + \frac{c}{p^2} \\ y = -2p^3 + \frac{2c}{p} \end{cases} \quad (p \neq 0)$$

$p = 0 \Rightarrow y \equiv 0$ - singularno rješenje