GRUPA D Ispit 18.02.2011 EF

1. $$M \overset{I_K \leftrightarrow IV_K}{\sim} \begin{bmatrix} 1 & 4 & -3 & 4 \\ 0 & 1 & -1 & 1 \\ 2 & 2 & 2 & a \\ 3 & 9 & a & 9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 4 & -3 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & -6 & 8 & a-8 \\ 0 & -3 & a+9 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & -3 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & a-2 \\ 0 & 0 & a+6 & 0 \end{bmatrix}$$

$$\overset{III_v \cdot (a+6) - IV_v \cdot 2}{\sim} \begin{bmatrix} 1 & 4 & -3 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & a-2 \\ 0 & 0 & 0 & (a-2)(a+6) \end{bmatrix}$$

Diskusija:

1° $a = 2 \lor a = -6 \Rightarrow r(M) = 3$

2° $a \neq 2 \land a \neq -6 \Rightarrow r(M) = 4$

2. $$y = \frac{\ln x}{1 - \ln x}$$

Def područje: $x > 0 \land 1 - \ln x \neq 0$

$1 - \ln x = 0 \Rightarrow 1 = \ln x \Leftrightarrow \ln e = \ln x \Rightarrow x = e$

$x \in (0, +\infty) \land x \neq e \Rightarrow \boxed{x \in (0, e) \cup (e, +\infty)}$

Nule: $\ln x = 0 \Rightarrow x = 1$

Znaki

	0 – 1	1 – e	e – $+\infty$
y	$-$ 0	$+$	$-$

$$\lim_{x\to 0_+} \frac{\ln x}{1-\ln x} = \left| \begin{matrix} \ln x = t \\ t \to -\infty \end{matrix} \right| = \lim_{t\to -\infty} \frac{t}{1-t} \frac{/:t}{/:t}$$

$$= \lim_{t\to -\infty} \frac{1}{\frac{1}{t} - 1} = -1$$

$$\left.\begin{matrix} \lim\limits_{x\to e_-} \dfrac{\ln x}{1-\ln x} = \dfrac{1}{0_+} = +\infty \\ \lim\limits_{x\to e_+} \dfrac{\ln x}{1-\ln x} = \dfrac{1}{0_-} = -\infty \end{matrix}\right\} \text{V.A. } x = e$$

$$\lim_{x\to +\infty} \frac{\ln x}{1-\ln x} = \left| \begin{matrix} \ln x = t \\ t \to +\infty \end{matrix} \right| = \lim_{t\to +\infty} \frac{t}{1-t} \frac{/:t}{/:t}$$

$$= \lim_{t\to +\infty} \frac{1}{\frac{1}{t} - 1} = -1$$

H.A. $y = -1$ (K.A. nema)

$$y' = \frac{\frac{1}{x}(1-\ln x) - \ln x \cdot \left(-\frac{1}{x}\right)}{(1-\ln x)^2} = \frac{\frac{1}{x} - \frac{1}{x}\ln x + \frac{1}{x}\ln x}{(1-\ln x)^2}$$

$$y' = \frac{1}{x(1-\ln x)^2} \Rightarrow y' \neq 0 \Rightarrow \text{nema ekstrema}$$

$y' > 0 \Rightarrow$ funkcija je monotono rastuća

$$y'' = \frac{-\left[(1-\ln x)^2 + \cancel{x} \cdot 2(1-\ln x) \cdot \left(-\frac{1}{\cancel{x}}\right)\right]}{x^2 (1-\ln x)^4}$$

$$y'' = -\frac{(1-\ln x)(1-\ln x - 2)}{x^2(1-\ln x)^4} = \frac{\ln x + 1}{x^2(1-\ln x)^3}$$

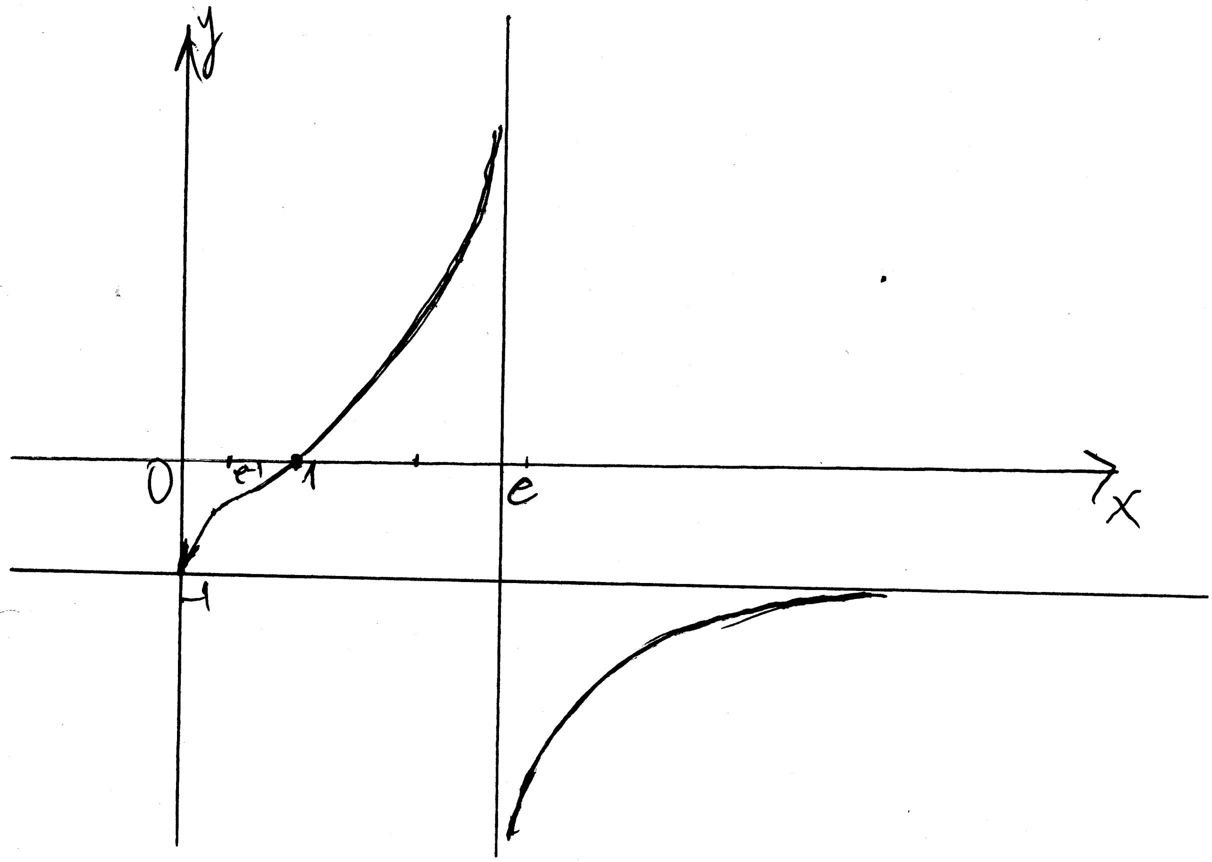
$$y'' = 0 \Rightarrow \ln x + 1 = 0 \Rightarrow \ln x = -1 \Rightarrow \underline{x = e^{-1}}$$

	0 — e^{-1}	e^{-1} — e	e — $+\infty$
$\ln x+1$	$-$ 0	$+$	$+$
$(1-\ln x)^3$	$+$	$+$ 0	$-$
y''	$-$	$+$	$-$
y	$\cap$	$\cup$	$\cap$

P.T.

$$y(e^{-1}) = \frac{\ln e^{-1}}{1-\ln e^{-1}} = \frac{-1}{1+1} = -\frac{1}{2}$$

$P\left(\frac{1}{e}, -\frac{1}{2}\right)$ - prevojna točka



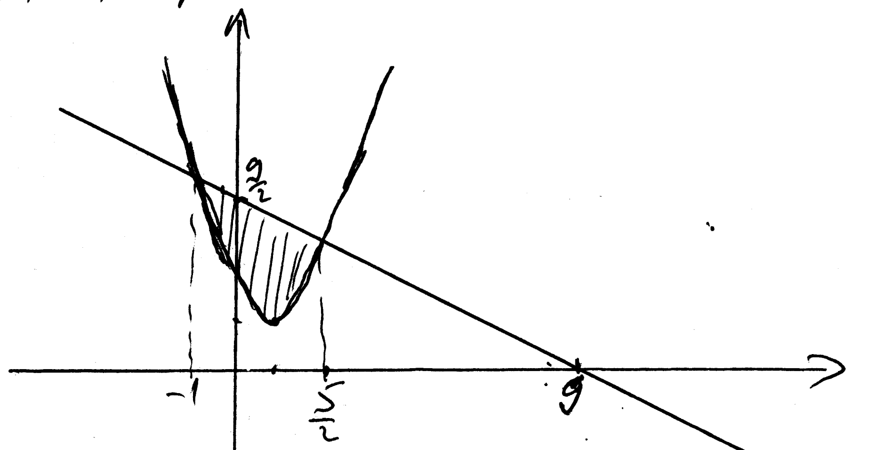
3 $\quad y = x^2-2x+2 = (x-1)^2+1$ - parabola sa tjemenom $T(1,1)$

$y(0) = 2$

$x+2y-9=0$

$x+2y=9$

$\frac{x}{9} + \frac{y}{\frac{9}{2}} = 1$



Zajedničke točke:

$x+2(x^2-2x+2)-9=0$

$2x^2-3x-5=0$

$x_1=-1, \; x_2=\frac{5}{2}$

$$P = \int_{-1}^{\frac{5}{2}} \left[\frac{-x+9}{2} - (x^2-2x+2)\right] dx = \dots = \frac{343}{48}$$

4. $z = x^2y^2 - 2xy^2 - 6x^2y + 12xy$

$z'_x = 2xy^2 - 2y^2 - 12xy + 12y = 2y^2(x-1) - 12y(x-1)$
$z'_y = 2x^2y - 4xy - 6x^2 + 12x = 2xy(x-2) - 6x(x-2)$

$z'_x = (2y^2 - 12y)(x-1) = 2y(y-6)(x-1)$
$z'_y = (2xy - 6x)(x-2) = 2x(y-3)(x-2)$

$z'_x = 0 \Rightarrow y = 0 \vee y = 6 \vee x = 1$
$z'_y = 0 \Rightarrow x = 0 \vee y = 3 \vee x = 2$

Stacionarne tačke: $M_1(1,3)$, $M_2(0,0)$, $M_3(0,6)$, $M_4(2,0)$, $M_5(2,6)$

$z''_{xx} = 2y^2 - 12y = 2y(y-6)$
$z''_{xy} = 4xy - 4y - 12x + 12 = 4(y-3)(x-1)$
$z''_{yy} = 2x^2 - 4x = 2x(x-2)$

Za tačku M_1: $A = 2\cdot 9 - 12\cdot 3 = -18$, $B = 12 - 12 - 12 + 12 = 0$
$C = 2 - 4 = -2$
$D = 36$, $A < 0 \Rightarrow$ u M_1 je maksimum

Za tačku M_2: $A = C = 0$, $B = 12 \Rightarrow D = -12^2 < 0$
U tački M_2 nije ekstrem

Za tačku M_3: $A = 72 - 72 = 0$, $B = -12$, $C = 0 \Rightarrow D < 0$
U tački M_3 nije ekstrem

Za tačku M_4: $A = 0$, $B = -12$, $C = 0 \Rightarrow D < 0$
U tački M_4 nije ekstrem

Za tačku M_5: $A = C = 0$, $B = 12 \Rightarrow D < 0$
U tački M_5 nije ekstrem