(#) Riješiti sistem jednačina

$$\begin{aligned} 2x_1 - x_2 + x_3 + 2x_4 + 3x_5 &= 2\\ 6x_1 - 3x_2 + 2x_3 + 4x_4 + 5x_5 &= 3\\ 6x_1 - 3x_2 + 4x_3 + 8x_4 + 13x_5 &= 9\\ 4x_1 - 2x_2 + x_3 + x_4 + 2x_5 &= 1 \end{aligned}$$

.

Rj.

$$\begin{aligned} 2x_1 - x_2 + x_3 + 2x_4 + 3x_5 &= 2 && (a)\\ 6x_1 - 3x_2 + 2x_3 + 4x_4 + 5x_5 &= 3 && (b)\\ 6x_1 - 3x_2 + 4x_3 + 8x_4 + 13x_5 &= 9 && (c)\\ 4x_1 - 2x_2 + x_3 + x_4 + 2x_5 &= 1 && (d) \end{aligned}$$

Izaberimo proizvoljnu nepoznatu koje ćemo se riješiti.

Riješimo se nepoznate x_3.

$$\begin{aligned} (a)-(d):&\quad -2x_1 + x_2 + x_4 + x_5 = 1 && (I)\\ (b)-2(d):&\quad -2x_1 + x_2 + 2x_4 + x_5 = 1 && (II)\\ (c)+(d)\cdot(-4):&\quad -18x_1 + 5x_2 + 3x_4 + 5x_5 = 5 && (III) \end{aligned}$$

Riješimo se nepoznate x_2.

$$\left.\begin{aligned} (III)+(II)\cdot(-5):&\quad -8x_1 - 2x_4 = 0\\ (II)-(I):&\quad x_4 = 0 \end{aligned}\right\} \Rightarrow x_1 = 0$$

$$\begin{aligned} x_2 + x_5 &= 1\\ 5x_2 + 5x_5 &= 5 \end{aligned}$$

Jednu promjenjivu iz ovog sistema možemo uzeti proizvoljno, pa neka je to $x_2 = s$. $\Rightarrow$ $x_5 = 1-s$

Trebamo još odrediti x_3

$$2\underset{0}{\underset{\parallel}{x_1}} - x_2 + x_3 + 2\underset{0}{\underset{\parallel}{x_4}} + 3x_5 = 2 \quad\Rightarrow\quad -s + x_3 + 3 - 3s = 2$$

$$x_3 = 4s - 1$$

Rješenje sistema jednačina je $(0,\ s,\ 4s-1,\ 0,\ 1-s)$.

Da smo za proizvoljnu promjenjivu uzeli nepoznatu x_5 rješenje sistema bi bilo $(0,\ 1-s,\ 3-4s,\ 0,\ s)$.
ili da smo uzeli x_1
rj. $(s,\ 0,\ -1-8s,\ 0,\ 1+2s)$

(#) Ispitati f-ju $y=\dfrac{x^2+ax+b}{x+6}$ i nacrtati joj grafik ako su nule f-je $x_{1,2}=\dfrac{-7\pm\sqrt{21}}{2}$.

Rj:

$y=0$ akko $x^2+ax+b=0$

$D=a^2-4b$

$x_{1,2}=\dfrac{-a\pm\sqrt{a^2-4b}}{2}$

Kako je dato da je $x_{1,2}=\dfrac{-7\pm\sqrt{21}}{2}$ možemo zaključiti da je

$a=7$, $49-4b=21$

$-4b=-28$

$b=7$

$$y=\frac{x^2+7x+7}{x+6}$$

definiciono područje

$x+6\neq 0$

$x\neq -6$

$\mathcal{D}$: $x\in\mathbb{R}\setminus\{-6\}$

$x\in(-\infty,-6)\cup(-6,+\infty)$

nule, presjek sa y-osom, znak

nule f-je dobijemo za $x_1=\dfrac{-7-\sqrt{21}}{2}\approx -5{,}79$

$x_2=\dfrac{-7+\sqrt{21}}{2}\approx -1{,}21$

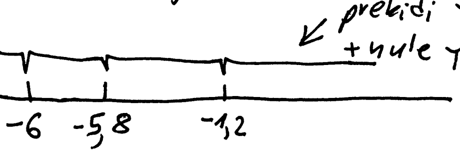
nule f-je su $(-5{,}79;0)$ i $(-1{,}21;0)$

$f(0)=\dfrac{7}{6}\approx 1{,}17$ Presjek sa y-osom je $(0;1{,}17)$

parnost (neparnost), periodičnost

$\mathcal{D}$ nije simetrično $\Rightarrow$ f-ja nije ni parna ni neparna

f-ja nije periodična



x	$(-\infty,-6)$	$(-6,-5{,}8)$	$(-5{,}8,-1{,}2)$	$(-1{,}2,+\infty)$
x^2+7x+7	+	+	−	+
$x+6$	−	+	+	+
y	−	+	−	+

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

Za $x=-6$ f-ja ima prekid

$\lim\limits_{x\to-6-0} f(x)=\lim\limits_{x\to-6-0}\dfrac{x^2+7x+7}{x+6}=\dfrac{\text{pozitivan broj}}{-6-0+6}=-\infty$

$\lim\limits_{x\to-6+0} f(x)=\lim\limits_{x\to-6+0}\dfrac{x^2+7x+7}{x+6}=\dfrac{\text{pozitivan broj}}{+0}=+\infty$

$\Rightarrow$ $x=-6$ je V. A. (i sa desne i sa lijeve strane)

$\lim\limits_{x\to\mp\infty} f(x)=\lim\limits_{x\to\mp\infty}\dfrac{x^2+7x+7\ /:x}{x+6\ /:x}=\mp\infty$ $\Rightarrow$ f-ja nema H. A.

Tražimo kosu asimptotu u obliku $y=kx+n$

$k=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to\infty}\dfrac{x^2+7x+7\ /:x^2}{x^2+6x\ /:x^2}=1$

$n=\lim\limits_{x\to\infty}\left[f(x)-k\cdot x\right]=\lim\limits_{x\to\infty}\left(\dfrac{x^2+7x+7}{x+6}-x\right)=\lim\limits_{x\to\infty}\dfrac{\cancel{x^2}+7x+7-\cancel{x^2}-6x}{x+6}=$

$=\lim\limits_{x\to\infty}\dfrac{x+7\ /:x}{x+6\ /:x}=1$

$y=x+1$ je K. A.

poslije ovog koraka počinjemo skicirati grafik f-je.

rast i opadanje

$$y' = \left(\frac{x^2+7x+7}{x+6}\right)' = \frac{(2x+7)(x+6)-(x^2+7x+7)\cdot 1}{(x+6)^2} = \frac{2x^2+19x+42-x^2-7x-7}{(x+6)^2}$$

$$y' = \frac{x^2+12x+35}{(x+6)^2}$$

$y'=0$ akko

$x^2+12x+35=0$

$D=144-140=4$

$x_{1,2} = \frac{-12 \pm 2}{2}$

$x_1=-7 \quad x_2=-5$

prekidi y + nule y'

-7 -6 -5

x	$(-\infty,-7)$	$(-7,-6)$	$(-6,-5)$	$(-5,+\infty)$
y'	+	-	-	+
y	↗	↘	↘	↗
		max		min

rast i opadanje

$f(-7)=-7, \quad f(-5)=-3$

ekstremi f-je

Na osnovu tabele rasta i opadanja vidimo da f-ja ima maksimum u tački $(-7,-7)$ i minimum u tački $(-5,-3)$.

prevojne tačke, i intervali konveksnosti i konkavnosti

$$y'' = \frac{(2x+12)(x+6)^{\cancel{2}} - (x^2+12x+35)\cdot 2\cancel{(x+6)}}{(x+6)^{\cancel{4}\,3}} = 2\,\frac{(x+6)^2-(x^2+12x+35)}{(x+6)^3} =$$

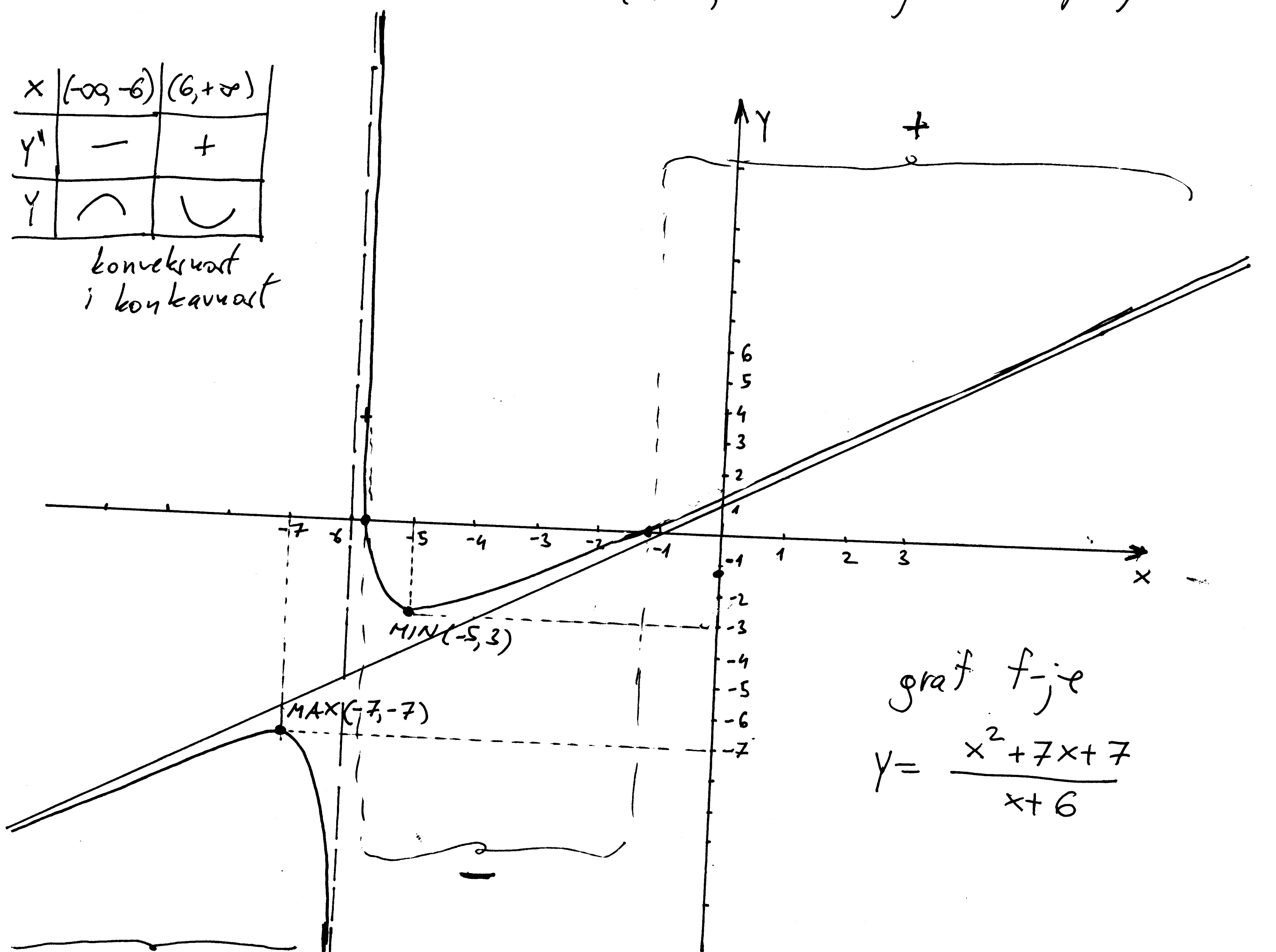
$$= 2\,\frac{x^2+\cancel{12x}+36-x^2-\cancel{12x}-35}{(x+6)^3} = \frac{2}{(x+6)^3}$$

$y'' \neq 0 \ \forall x$

f-ja nema prevojnu tačku

x	$(-\infty,-6)$	$(6,+\infty)$
y''	−	+
y	∩	∪

konveksnost i konkavnost



graf f-je

$$y = \frac{x^2+7x+7}{x+6}$$

(#) Izračunati integral $\int \frac{dx}{\sqrt[4]{x^3}(1+\sqrt[6]{x})}$.

Rj.

$$\int \frac{dx}{\sqrt[4]{x^3}(1+\sqrt[6]{x})} = \left| \begin{array}{l} x = t^{12} \qquad t = \sqrt[12]{x} \\ dx = 12t^{11}dt \\ \sqrt[4]{x^3} = \sqrt[4]{t^{36}} = t^9 \\ \sqrt[6]{x} = \sqrt[6]{t^{12}} = t^2 \end{array} \right| = \int \frac{12t^{11}dt}{t^9(1+t^2)} =$$

$$= 12\int \frac{t^2 \overset{+1-1}{}}{1+t^2}dt = 12\int \frac{t^2+1}{t^2+1}dt - 12\int \frac{dt}{1+t^2} =$$

$$= 12t - 12\,\mathrm{arctg}\,t + C = 12\sqrt[12]{x} - 12\,\mathrm{arctg}\sqrt[12]{x} + C$$

(#) Naći ekstreme f-je $z = x^4 + y^4 - 2(x-y)^2$.

Rj.

$z'_x = 4x^3 - 2\cdot 2(x-y)\cdot 1$

$= 4x^3 - 4x + 4y$

$z'_y = 4y^3 - 2\cdot 2(x-y)\cdot(-1)$

$= 4y^3 + 4x - 4y$

$4x^3 - 4x + 4y = 0 \quad /:4$

$4y^3 + 4x - 4y = 0 \quad /:4$

$x^3 - x + y = 0$

$+ \quad y^3 + x - y = 0$

$x^3 + y^3 = 0$

$x = -y$

$-4y^3 + 4y + 4y = 0 \quad /:4$

$2y - y^3 = 0$

$y(2 - y^2) = 0$

$y_1 = 0,\ y_2 = \sqrt{2},\ y_3 = -\sqrt{2}$

Stacionarne tačke su $M_1(0,0)$, $M_2(\sqrt{2}, -\sqrt{2})$ i $M_3(-\sqrt{2}, \sqrt{2})$

Nađimo druge parcijalne izvode

$\frac{\partial^2 z}{\partial x^2} = 12x^2 - 4$

$\frac{\partial^2 z}{\partial x \partial y} = 4$

$\frac{\partial^2 z}{\partial y^2} = 12y^2 - 4$

$M_1(0,0)$

$A = -4$

$B = 4$

$C = -4$

$D = AC - B^2 = 16 - 16 = 0$

Trebamo ispitati ponašanje f-ju u okolini tačke $(0,0)$

Neka su ε i η proizvoljni maleni brojevi (pozitivni ili negativni).

$\Delta z = z(0+\varepsilon, 0+\eta) - z(0,0) = z(\varepsilon, \eta) - 0 = \varepsilon^4 + \eta^4 - 2(\varepsilon - \eta)^2$

$= \varepsilon^4 - 2(\varepsilon - \eta)^2 + \eta^4 = \varepsilon^4 - 2(\varepsilon^2 - 2\varepsilon\eta + \eta^2) + \eta^4 =$

$= \varepsilon^4 - 2\varepsilon^2 + 4\varepsilon\eta - 2\eta^2 + \eta^4 = \ldots$ vratimo se na $\Delta z = \varepsilon^4 \eta^4 - 2(\varepsilon - \eta)^2$

za $\eta = 0$: $\Delta z = \varepsilon^4 - 2\varepsilon^2 = \varepsilon^2(\varepsilon^2 - 2) \longrightarrow$ za $\varepsilon^2 > 2$, $\Delta z > 0$; za $\varepsilon^2 < 2$, $\Delta z < 0$

Primijetimo; Δz je promjenjivog znaka pa f-ja u tački $M_1(0,0)$ nema ekstrema.

$M_2(\sqrt{2}, -\sqrt{2})$, $A = 20$, $B = 4$, $C = 20$, $D = AC - B^2 > 0$, $A > 0$

U tački $M_2(\sqrt{2}, -\sqrt{2})$ f-ja ima minimum $z_{min}(\sqrt{2}, -\sqrt{2}) = -8$

$M_3(-\sqrt{2}, \sqrt{2})$, $A = 20$, $B = 4$, $C = 20$, $D = AC - B^2 > 0$, $A > 0$

U tački $M_3(-\sqrt{2}, \sqrt{2})$ f-ja ima min $z_{min}(\sqrt{2}, -\sqrt{2}) = 4 + 4 - 2\cdot 4\cdot 2 = -8$