Univerzitet u Zenici
Fakultet za metalurgiju i materijale

14.09.2009.

Pismeni ispit iz predmeta Matematika 1

1. Riješiti matričnu jednačinu $XA^{-1} = B^{-1}$ ako su $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -4 \end{bmatrix}$ i $B = \begin{bmatrix} 2 & 1 & -1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}$.

2. Vektori $\vec{a} = (-1, -3, 1)$, $\vec{b} = (\lambda, 3, 4)$ i $\vec{c} = (-5, -9, 1)$ su ivice tetraedra. Odrediti parametar λ tako da zapremina tetraedra iznosi 8. Za vrijednost $\lambda = 6$ provjeriti da li su vektori $\vec{a}$, $\vec{b}$ i $\vec{c}$ komplanarni, pa ako jesu izraziti vektor $\vec{a}$ preko vektora $\vec{b}$ i $\vec{c}$.

3. Odredite parametar λ u jednačini prave $\frac{x-3}{1} = \frac{y-1}{\lambda} = \frac{z+2}{1}$ da bi se sjekla sa pravom $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-1}{-1}$ i u tom slučaju naći presječnu tačku i ugao između pravih.

4. Izračunati: $\lim\limits_{x \to 1} \frac{\sqrt[3]{x} - 1}{1 - x}$.

14.09.2009.

Pismeni ispit iz predmeta Matematika 2

1. Ispitati i grafički predstaviti funkciju: $y = xe^{\frac{1}{x}}$.

2. Izračunati: $\int_3^4 \frac{6x+8}{x^2+x-6}\,dx$.

3. Odrediti ekstremne vrijednosti funkcije $z = 6\sqrt{3}\,x^3 - 2y^3 + 6xy + \frac{2}{9}\sqrt{3}$.

4. Odrediti područje konvergencije reda $\sum\limits_{n=1}^{\infty} \frac{(x+2)^n}{n\,2^n}$.

(Ova stranica je ostavljena prazna)

Riješiti matričnu jednačinu $X \cdot A^{-1} = B^{-1}$ ako su

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -4 \end{bmatrix} \quad i \quad B = \begin{bmatrix} 2 & 1 & -1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}.$$

Rj.

$$X \cdot A^{-1} = B^{-1} \quad / \cdot A \text{ sa desne strane}$$

$$X \underbrace{A^{-1} \cdot A}_{I} = B^{-1} \cdot A$$

$$X = B^{-1} \cdot A$$

$$\det B = \begin{vmatrix} 2 & 1 & -1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{vmatrix} \underline{\underline{I_R - II_R}}$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 2 & 1 & -2 \\ -1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} = 0 + 1$$

$$\det B = 1$$

$$B^{-1} = \frac{1}{\det B} B_{kof}^{T}$$

$$B_{11} = (-1)^2 \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1 \qquad B_{21} = (-1)^3 \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = -1 \qquad B_{31} = (-1)^4 \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} = -1$$

$$B_{12} = (-1)^3 \begin{vmatrix} 2 & -2 \\ -1 & 1 \end{vmatrix} = 0 \qquad B_{22} = (-1)^4 \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = 1 \qquad B_{32} = (-1)^5 \begin{vmatrix} 2 & -1 \\ 2 & -2 \end{vmatrix} = 2$$

$$B_{13} = (-1)^4 \begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} = 1 \qquad B_{23} = (-1)^5 \begin{vmatrix} 2 & 1 \\ -1 & 0 \end{vmatrix} = -1 \qquad B_{33} = (-1)^6 \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = 0$$

$$B_{kof} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -1 \\ -1 & 2 & 0 \end{bmatrix}, \quad B_{kof}^{T} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix}, \quad B^{-1} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix}$$

$$X = B^{-1} \cdot A = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 1 & 1 & -4 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 7 \\ 5 & 4 & -7 \\ -1 & 1 & 3 \end{bmatrix}$$

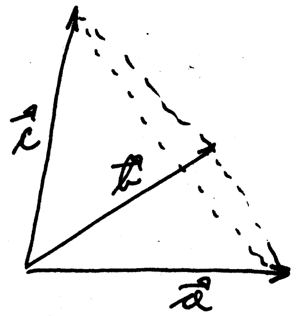
$$\begin{matrix} 2-3-1 & 0+3+2 & 2-3+0 \\ 3-2-1 & 0+2+2 & 3-2+0 \\ 4-1+4 & 0+1-8 & 4-1+0 \end{matrix}$$

$$X = \begin{bmatrix} -2 & 0 & 7 \\ 5 & 4 & -7 \\ -1 & 1 & 3 \end{bmatrix}$$

traženo rješenje

(#) Vektori $\vec{a}=(-1,-3,1)$, $\vec{b}=(\lambda,3,4)$ i $\vec{c}=(-5,-9,1)$ su ivice tetraedra. Odrediti parametar λ tako da zapremina tetraedra iznosi 8. Za vrijednost $\lambda=6$ provjeriti da li su vektori $\vec{a}$, $\vec{b}$ i $\vec{c}$ komplanarni, pa ako jesu izraziti vektor $\vec{a}$ preko vektora $\vec{b}$ i $\vec{c}$.

R:



$$V=\frac{1}{6}\left|(\vec{a}\times\vec{b})\cdot\vec{c}\right| = \left|\frac{1}{6}\begin{vmatrix}-1 & -3 & 1\\ \lambda & 3 & 4\\ -5 & -9 & 1\end{vmatrix}\right| \begin{array}{l} I_R - III_R \\ \hline II_R - III_R\cdot 4\end{array}$$

$$=\left|\frac{1}{6}\begin{vmatrix}4 & 6 & 0\\ \lambda+20 & 39 & 0\\ -5 & -9 & 1\end{vmatrix}\right| = \left|\frac{1}{6}\begin{vmatrix}4 & 6\\ \lambda+20 & 39\end{vmatrix}\right| =$$

$$=\left|\frac{1}{6}(156-6\lambda-120)\right| = \left|\frac{1}{6}(36-6\lambda)\right| = \left|\frac{1}{6}\cdot 6(6-\lambda)\right|$$

$V=|6-\lambda|$

$V=8$ $\Rightarrow$ $\lambda=-2$ Za $\lambda=-2$ zapremina tetraedra iznosi 8.

Za vrijednost $\lambda=6$ zapremina tetraedra je 0 pa su vektori $\vec{a}=(-1,-3,1)$, $\vec{b}=(6,3,4)$ i $\vec{c}=(-5,-9,1)$ komplanarni.

$\vec{a}=\alpha\vec{b}+\beta\vec{c}$

$(-1,-3,1)=(6\alpha,3\alpha,4\alpha)+(-5\beta,-9\beta,\beta)$

$$\begin{aligned} 6\alpha-5\beta&=-1\\ 3\alpha-9\beta&=-3 \quad /:3\\ 4\alpha+\beta&=1\end{aligned}$$

$$\begin{aligned} 6\alpha-5\beta&=-1\\ \alpha-3\beta&=-1\\ 4\alpha+\beta&=1\end{aligned}$$

$\alpha=3\beta-1$

$6\alpha-5\beta=-1$

$6(3\beta-1)-5\beta=-1$

$18\beta-6-5\beta=-1$

$13\beta=5$

$\beta=\frac{5}{13}$

$\alpha=\frac{15}{13}-\frac{13}{13}$

$\alpha=\frac{2}{13}$

$\vec{a}=\frac{2}{13}\vec{b}+\frac{5}{13}\vec{c}$ vektor $\vec{a}$ izražen preko vektora $\vec{b}$ i $\vec{c}$.

(#) Odrediti λ u jednačini prave $\frac{x-3}{1}=\frac{y-1}{\lambda}=\frac{z+2}{1}$ da bi se sjekla sa pravom $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{-1}$ i u tom slučaju naći presječnu tačku i ugao između pravih.

Rj:

$a: \ \frac{x-3}{1}=\frac{y-1}{\lambda}=\frac{z+2}{1}, \quad \vec{p}_a=(1,\lambda,1), \quad x_1=3, \ y_1=1, \ z_1=-2$

$b: \ \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{-1}, \quad \vec{p}_b=(2,1,-1), \quad x_2=1, \ y_2=2, \ z_2=1$

Potreban uslov da se prave sijeku: $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0.$

$$\begin{vmatrix} -2 & 1 & 3 \\ 1 & \lambda & 1 \\ 2 & 1 & -1 \end{vmatrix} \ \frac{I_R+III_R\cdot 3}{II_R+III_R} \ \begin{vmatrix} 4 & 4 & 0 \\ 3 & \lambda+1 & 0 \\ 2 & 1 & -1 \end{vmatrix} = (-1)\begin{vmatrix} 4 & 4 \\ 3 & \lambda+1 \end{vmatrix} = (-1)(4\lambda+4-12)=(-1)(4\lambda-8)$$

$(-1)(4\lambda-8)=0$

$\lambda=2$

Za vrijednost $\lambda=2$ prave a i b se sijeku.

a:

$\frac{x-3}{1}=\frac{y-1}{2}=\frac{z+2}{1} \quad (=t)$

$x-3=t \qquad x=t+3$

$y-1=2t \qquad y=2t+1$

$z+2=t \qquad z=t-2$

b:

$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-1}{-1} \quad (=s)$

$x-1=2s \qquad x=2s+1$

$y-2=s \qquad y=s+2$

$z-1=-s \qquad z=-s+1$

$t+3=2s+1$

$2t+1=s+2$

$t-2=-s+1$

$t-2s=-2 \quad /\cdot 2$

$2t-s=1$

$t+s=3 \quad /\cdot 2$

$2t-4s=-4 \quad (1)$

$2t-s=1 \quad (2)$

$2t+2s=6 \quad (3)$

$(1)-(3): \ -6s=-10$

$(2)-(3): \ -3s=-5$

$s=\frac{5}{3}$

$t=2s-2=\frac{10}{3}-\frac{6}{3}=\frac{4}{3} \qquad x=\frac{4}{3}+3=\frac{13}{3}, \quad y=\frac{8}{3}+1=\frac{11}{3}, \quad z=\frac{4}{3}-2=-\frac{2}{3}$

Presječna tačka pravih je $M\left(\frac{13}{3}, \frac{11}{3}, -\frac{2}{3}\right)$.

$\vec{p}_a\cdot\vec{p}_b=(1,2,1)\cdot(2,1,-1)=2+2-1=3$

$|\vec{p}_a|=\sqrt{1+4+1}=\sqrt{6}, \quad |\vec{p}_b|=\sqrt{4+1+1}=\sqrt{6}$

$\vec{p}_a\cdot\vec{p}_b=|\vec{p}_a|\cdot|\vec{p}_b|\cdot\cos\measuredangle(\vec{p}_a,\vec{p}_b)$

$\Rightarrow \cos\measuredangle(\vec{p}_a,\vec{p}_b)=\frac{\vec{p}_a\cdot\vec{p}_b}{|\vec{p}_a|\cdot|\vec{p}_b|}=\frac{3}{6}=\frac{1}{2} \Rightarrow \measuredangle(\vec{p}_a,\vec{p}_b)=60^\circ$ ugao između pravih

(#) Izračunati $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x}-1}{1-x}$.

Rj. $a^3-b^3=(a-b)(a^2+ab+b^2)$

$$\left(\sqrt[3]{x}-1\right)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)=\left(\sqrt[3]{x}\right)^3-1^3=x-1$$

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{1-x}\left(\frac{0}{0}\right)=-\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{x-1}\,\frac{\cdot\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}{\cdot\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}=-\lim_{x\to 1}\frac{\cancel{(x-1)}}{\cancel{(x-1)}\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}$$

$$=-\lim_{x\to 1}\frac{1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}=\frac{-1}{\sqrt[3]{1}+\sqrt[3]{1}+1}=-\frac{1}{3}$$

Ispitati i grafički predstaviti f-ju $y = x e^{\frac{1}{x}}$.

Rj: definiciono područje

$x \neq 0$, $\mathcal{D}: x \in \mathbb{R} \setminus \{0\}$

parnost, neparnost, periodičnost

$f(-x) = -x e^{\frac{1}{-x}} = -x e^{-\frac{1}{x}}$

f-ja nije ni parna ni neparna

f-ja nije periodična

nule, presjek sa Y-osom, znak f-je

$x e^{\frac{1}{x}} = 0$

$x = 0$ ili $e^{\frac{1}{x}} = 0$

nije definisano $\quad e^{x} \neq 0 \ \forall x \in \mathbb{R}$

f-ja nema nulu

$f(0)$ nije definisano

f-ja ne siječe Y-osu

$e^{\frac{1}{x}} > 0 \ \forall x \in \mathcal{D}$

X	$(-\infty, 0)$	$(0, +\infty)$
Y	$-$	$+$

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

$x = 0$ f-ja ima prekid

$\left(-\frac{1}{x}\right)' = (-x^{-1})'$

$$\lim_{x \to -0} f(x) = \lim_{x \to -0} x e^{\frac{1}{x}} = (-0) \cdot e^{\frac{1}{-0}} = (-0) \cdot e^{-\infty} = \frac{-0}{e^{\infty}} = \frac{-0}{\infty} = 0$$

$$\lim_{x \to +0} f(x) = \lim_{x \to +0} x e^{\frac{1}{x}} (= 0 \cdot \infty) = \lim_{x \to +0} \frac{x}{e^{-\frac{1}{x}}} \left(= \frac{0}{0}\right) \overset{LoP_0}{=\!=} \lim_{x \to +0} \frac{1}{e^{-\frac{1}{x}} \cdot \frac{1}{x^2}} = \lim_{x \to +0} \frac{x^2}{e^{-\frac{1}{x}}} = \frac{}{\infty}$$

pokušat ćemo na drugi način:

$$\lim_{x \to +0} f(x) = \lim_{x \to +0} x e^{\frac{1}{x}} (= 0 \cdot \infty) = \lim_{x \to 0+} \frac{e^{\frac{1}{x}}}{x^{-1}} \left(= \frac{\infty}{\infty}\right) \overset{LoP_0}{=\!=} \lim_{x \to 0+} \frac{e^{\frac{1}{x}} \cdot \left(\frac{1}{x}\right)'}{\left(\frac{1}{x}\right)'} = e^{\frac{1}{0+}} = \infty$$

$\Rightarrow x = 0$ je $V_0 A_0$

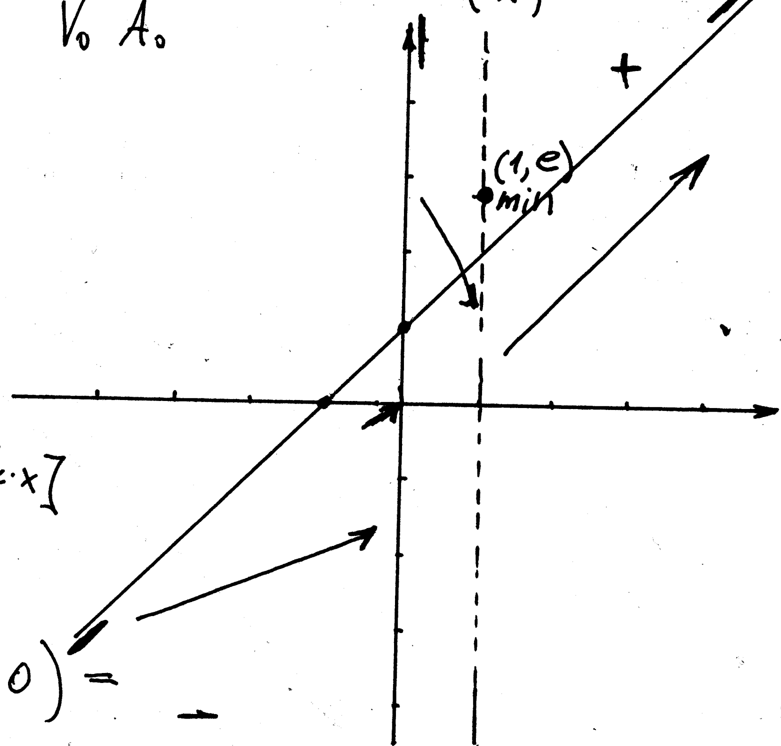
$$\left.\begin{array}{l} \lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} x e^{\frac{1}{x}} = -\infty \cdot 1 = -\infty \\ \lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} x e^{\frac{1}{x}} = +\infty \cdot 1 = \infty \end{array}\right\} \Rightarrow$$

$\Rightarrow$ f-ja nema $H_0 A_0$

$y = kx + n$, $\quad k = \lim\limits_{x \to \infty} \frac{f(x)}{x}$, $\quad n = \lim\limits_{x \to \infty} [f(x) - k \cdot x]$

$$k = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} e^{\frac{1}{x}} = e^0 = 1$$

$$n = \lim_{x \to \infty} [x e^{\frac{1}{x}} - x] = \lim_{x \to \infty} x (e^{\frac{1}{x}} - 1) \ (= \infty \cdot 0) =$$



$$= \lim_{x\to\infty} \frac{e^{\frac{1}{x}}-1}{\frac{1}{x}} \left(=\frac{0}{0}\right) \overset{L.P.}{=} \lim_{x\to\infty} \frac{e^{\frac{1}{x}}\cdot\cancel{\left(\frac{1}{x}\right)'}}{\cancel{\left(\frac{1}{x}\right)'}} = \lim_{x\to\infty} e^{\frac{1}{x}} = e^0 = 1$$

$Y = x + 1$ je K. A.

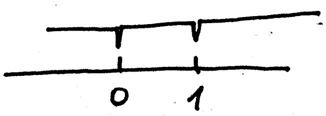
rast i opadanje

$$Y' = \left(x e^{\frac{1}{x}}\right)' = e^{\frac{1}{x}} + x\cdot e^{\frac{1}{x}}\cdot\left(x^{-1}\right)' = e^{\frac{1}{x}} + x e^{\frac{1}{x}}\cdot\left(-x^{-2}\right) = e^{\frac{1}{x}}\left(1 + x\cdot\left(-\frac{1}{x^2}\right)\right)$$

$$Y' = e^{\frac{1}{x}}\left(1 - \frac{1}{x}\right)$$

$Y' = 0$ akko $1 - \frac{1}{x} = 0$

$x = 1$



prekidi Y + nule Y'

x	$(-\infty, 0)$	$(0, 1)$	$(1, +\infty)$
Y'	+	−	+
Y	↗	↘	↗

MIN

rast i opadanje

ekstremi f-je

na osnovu tabele rasta i opadanja f-ja ima minimum u tački $(1, f(1))$. $f(1) = 1\cdot e^{\frac{1}{1}} = e$ $\quad f_{min}(1) = e$ $\quad (1, e)$

$e \approx 2{,}71$

prevojne tačke i intervali konveksnosti i konkavnosti

$$Y'' = \left(e^{\frac{1}{x}}\left(1-\frac{1}{x}\right)\right)' = e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right)\left(1-\frac{1}{x}\right) + e^{\frac{1}{x}}\cdot\left(-\left(-\frac{1}{x^2}\right)\right)$$

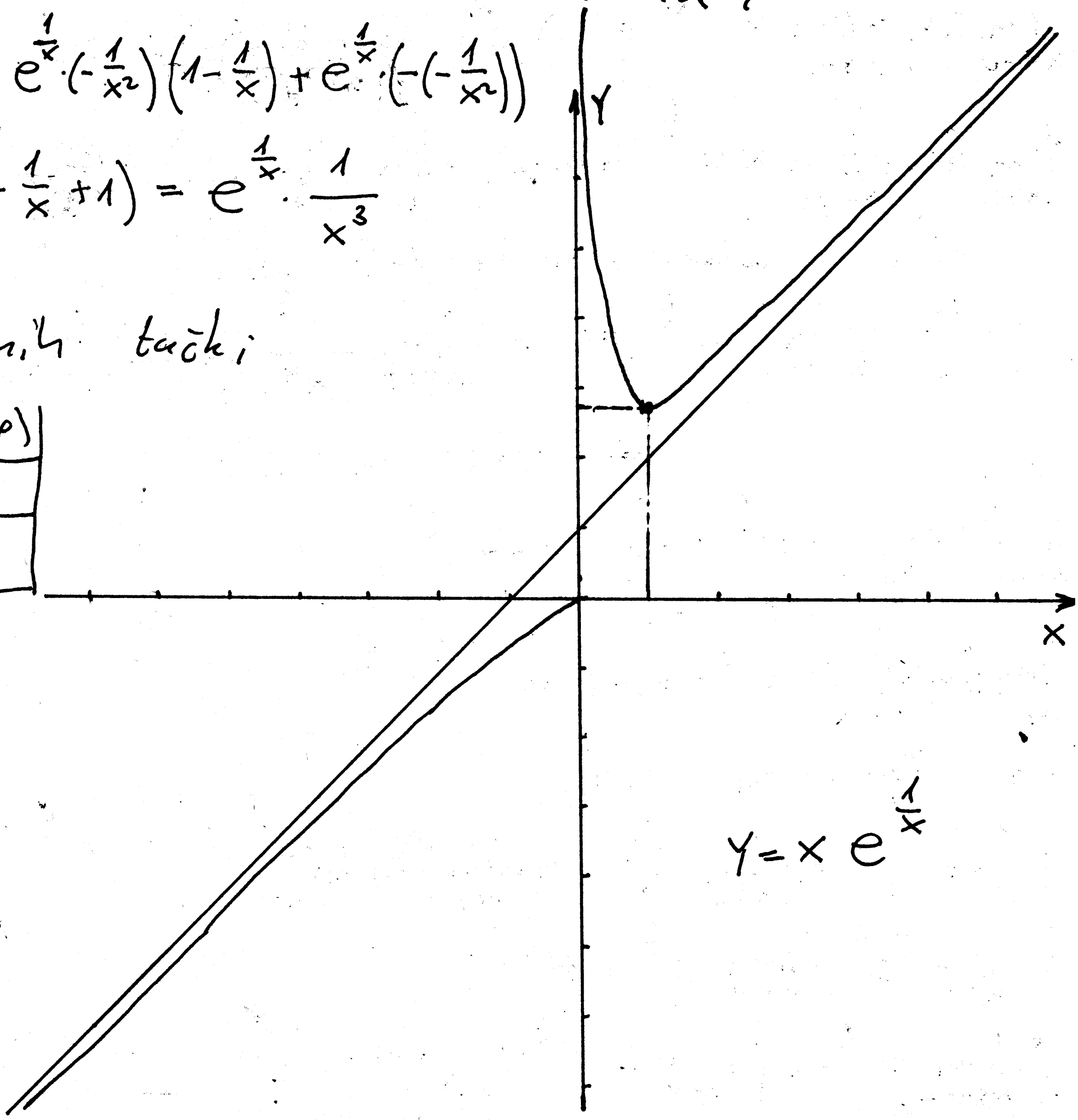
$$= e^{\frac{1}{x}}\cdot\frac{1}{x^2}\left(-1+\frac{1}{x}+1\right) = e^{\frac{1}{x}}\cdot\frac{1}{x^3}$$

$Y'' \neq 0 \quad \forall x \in D$

nema prevojnih tački

x	$(-\infty, 0)$	$(0, +\infty)$
Y''	−	+
Y	∩	∪

grafik



Izračunati: $\int_3^4 \frac{6x+8}{x^2+x-6}\,dx$

Rj.

$$\frac{6x+8}{x^2+x-6} = \frac{A}{x-2} + \frac{B}{x+3} \quad /(x-2)(x+3)$$

$D = 1+24 = 25$

$x_{1,2} = \frac{-1 \pm 5}{2}$

$x_1 = -3 \quad x_2 = 2$

$x^2+x-6 = (x-2)(x+3)$

$$6x+8 = A(x+3) + B(x-2)$$

$$6x+8 = (A+B)x + (3A-2B)$$

$$\begin{aligned} A+B &= 6 \quad /\cdot 2 \\ 3A-2B &= 8 \end{aligned}$$

$$\begin{aligned} 2A+2B &= 12 \\ +\ 3A-2B &= 8 \\ \hline 5A &= 20 \\ A &= 4 \end{aligned}$$

$A+B = 6$

$4+B = 6$

$B = 2$

$$\int \frac{6x+8}{x^2+x-6}\,dx = \int\left(\frac{4}{x-2} + \frac{2}{x+3}\right)dx = 4\int\frac{dx}{x-2} + 2\int\frac{dx}{x+3} =$$

$$= 4\ln|x-2| + 2\ln|x+3| + C$$

$$\int_3^4 \frac{6x+8}{x^2+x-6}\,dx = 4\ln|x-2|\Big|_3^4 + 2\ln|x+3|\Big|_3^4 = 4(\ln 2 - \underbrace{\ln 1}_{=0}) +$$

$$+ 2(\ln 7 - \ln 6) = 4\ln 2 + 2\ln\frac{7}{6} = \ln 2^4 + \ln\left(\frac{7}{6}\right)^2$$

$$= \ln\frac{7^2}{2^2\cdot 3^2}\cdot 2^4 = \ln\frac{49\cdot 4}{9} = \ln\frac{196}{9}$$

$$\int_3^4 \frac{6x+8}{x^2+x-6}\,dx = \ln\frac{196}{9} \quad \text{traženo rješenje}$$

(#) Odrediti ekstremne vrijednosti f-je

$$z = 6\sqrt{3}\,x^3 - 2y^3 + 6xy + \frac{2}{9}\sqrt{3}.$$

Rj.

$$\frac{\partial z}{\partial x} = 18\sqrt{3}\,x^2 + 6y$$

$$\frac{\partial z}{\partial y} = -6y^2 + 6x$$

$$18\sqrt{3}\,x^2 + 6y = 0 \quad /:6$$
$$-6y^2 + 6x = 0 \quad /:6$$

$$3\sqrt{3}\,x^2 + y = 0$$
$$-y^2 + x = 0$$

$$y = -3\sqrt{3}\,x^2$$
$$-y^2 + x = 0$$

$$-(-3\sqrt{3}\,x^2)^2 + x = 0$$

$$-9\cdot 3x^4 + x = 0$$

$$x - 27x^4 = 0$$

$$x(1 - 27x^3) = 0$$

$$x(1-3x)(1+3x+9x^2) = 0$$

$$x_1 = 0 \qquad x_2 = \frac{1}{3}$$

$$y = -3\sqrt{3}\,x^2$$

$$x_1 = 0 \Rightarrow y_1 = 0$$

$$x_2 = \frac{1}{3} \Rightarrow y_2 = -\cancel{3}\sqrt{3}\cdot\frac{1}{\cancel{3}\cdot 3} = -\frac{\sqrt{3}}{3}$$

Stacionarne tačke su $M_1(0,0)$ i $M_2\left(\frac{1}{3}, -\frac{\sqrt{3}}{3}\right)$

$$\frac{\partial^2 z}{\partial x^2} = 36\sqrt{3}\,x \qquad D = AC - B^2$$

$$\frac{\partial^2 z}{\partial x \partial y} = 6$$

$$\frac{\partial^2 z}{\partial y^2} = -12y$$

$M_1(0,0)$, $A = 0$, $B = 6$, $C = 0$, $D = -36 < 0$

U tački $M_1(0,0)$ f-ja z nema ekstrem

$M_2\left(\frac{1}{3}, -\frac{\sqrt{3}}{2}\right)$, $A = 12\sqrt{3}$, $B = 6$, $C = 4\sqrt{3}$

$D = 18\cdot 3 - 36 > 0$ f-ja ima ekstrem

$A > 0 \Rightarrow$ f-ja ima minimum

$$z_{min}\left(\frac{1}{3}, -\frac{\sqrt{3}}{3}\right) = \cancel{6}^{\,2}\sqrt{3}\cdot\frac{1}{\cancel{3}\cdot 9} + 2\cdot\frac{\cancel{3}\cdot\sqrt{3}}{\cancel{3}\cdot 9} + 6\cdot\frac{1}{3}\cdot\left(-\frac{\sqrt{3}}{3}\right) + \frac{2}{9}\sqrt{3} =$$

$$= \frac{2}{9}\sqrt{3} + \frac{2}{9}\sqrt{3} - \frac{6}{9}\sqrt{3} + \frac{2}{9}\sqrt{3} = 0$$

F-ja z ima minimum u tački $\left(\frac{1}{3}, -\frac{\sqrt{3}}{3}\right)$ čija je vrijednost 0.

(#) Odrediti područje konvergencije reda $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n \cdot 2^n}$.

Rj. Za stepeni red $\sum_{n=0}^{\infty} a_n (x-a)^n$ postoji broj R takav da za $\forall x$ $|x-a|<R$ red KV a za $\forall x$ $|x-a|>R$ red DV. Interval $(R-a, R+a)$ zove se interval konvergencije a R poluprečnik intervala KV reda. R se određuje po formuli

$\frac{1}{R} = \limsup_{n\to\infty} \sqrt[n]{|a_n|}$ ili po formuli $R = \lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right|$ ako ovaj limes postoji.

$$\sum_{n=1}^{\infty} \frac{(x+2)^n}{n \cdot 2^n} = \sum_{n=1}^{\infty} \frac{1}{n2^n}(x+2)^n, \quad a_n = \frac{1}{n2^n}, \quad a = -2$$

$$R = \lim_{n\to\infty} \left| \frac{\frac{1}{n2^n}}{\frac{1}{(n+1)2^{n+1}}} \right| = \lim_{n\to\infty} \frac{(n+1)\cancel{2^n}\cdot 2}{n \cdot \cancel{2^n}} = 2\lim_{n\to\infty}\left(1+\frac{1}{n}\right) = 2$$

$R-a = 2-(-2) = 4$ Za $x \in (0,4)$ red KV.

$R+a = 2+(-2) = 0$

Ispitajmo još KV reda u graničnim tačkama.

za $x=0$ imamo red $\sum_{n=1}^{\infty} \frac{2^n}{n \cdot 2^n} = \sum_{n=1}^{\infty} \frac{1}{n}$

Ovo je harmoniski red i ovaj red DV.

Za $x=4$ imamo red $\sum_{n=1}^{\infty} \frac{6^n}{n \cdot 2^n}$

$\boxed{\lim_{n\to\infty} \sqrt[n]{n} = 1}$

Cauchyjev korjen kriterij

$$q = \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \frac{6}{\sqrt[n]{n}\, 2} = \frac{6}{2} = 3 > 1 \text{ red DV}$$

Područje Konvergencije reda je $(0,4)$.