

Riješiti sistem jednačina i diskutovati rješenja sistema u zavisnosti od parametra λ :

$$\begin{aligned} x - y - \lambda z &= 1 \\ (\lambda+1)y + (\lambda-1)z &= 0 \\ (\lambda+1)x - (\lambda+1)z &= 1 \end{aligned}$$

Rj. $D = \begin{vmatrix} 1 & -1 & -\lambda \\ 0 & \lambda+1 & \lambda-1 \\ \lambda+1 & 0 & -(\lambda+1) \end{vmatrix} \stackrel{III_k + I_k}{=} \begin{vmatrix} 1 & -1 & 1-\lambda \\ 0 & \lambda+1 & \lambda-1 \\ \lambda+1 & 0 & 0 \end{vmatrix} = (\lambda+1) \begin{vmatrix} -1 & -(\lambda-1) \\ \lambda+1 & \lambda-1 \end{vmatrix} =$

$$= (\lambda+1)(\lambda-1) \begin{vmatrix} -1 & -1 \\ \lambda+1 & 1 \end{vmatrix} = \lambda(\lambda-1)(\lambda+1)$$

$$D_x = \begin{vmatrix} 1 & -1 & -\lambda \\ 0 & \lambda+1 & \lambda-1 \\ 1 & 0 & -(\lambda+1) \end{vmatrix} \stackrel{III_v - I_v}{=} \begin{vmatrix} 1 & -1 & -\lambda \\ 0 & \lambda+1 & \lambda-1 \\ 0 & 1 & -1 \end{vmatrix} \stackrel{-1 + \lambda+1}{=} \begin{vmatrix} \lambda+1 & \lambda-1 \\ 1 & -1 \end{vmatrix} = \lambda-1 - \lambda+1 = -2\lambda$$

$$D_y = \begin{vmatrix} 1 & 1 & -\lambda \\ 0 & 0 & \lambda-1 \\ \lambda+1 & 1 & -(\lambda+1) \end{vmatrix} = -(\lambda-1) \begin{vmatrix} 1 & 1 \\ \lambda+1 & 1 \end{vmatrix} = -(\lambda-1)(1-\lambda-1) = \lambda(\lambda-1)$$

$$D_z = \begin{vmatrix} 1 & -1 & 1 \\ 0 & \lambda+1 & 0 \\ \lambda+1 & 0 & 1 \end{vmatrix} = (\lambda+1) \begin{vmatrix} 1 & 1 \\ \lambda+1 & 1 \end{vmatrix} = -\lambda(\lambda+1)$$

$D=0$ ako $\lambda=0$ ili $\lambda=1$ ili $\lambda=-1$

Diskusija

1° $\lambda \neq 0$; $\lambda \neq 1$; $\lambda \neq -1$ sistem ima jedinstveno rješenje

$$x = \frac{D_x}{D} = \frac{-2\lambda}{\lambda(\lambda-1)(\lambda+1)} = \frac{-2}{(\lambda-1)(\lambda+1)}, \quad y = \frac{D_y}{D} = \frac{1}{\lambda+1}, \quad z = \frac{D_z}{D} = \frac{-1}{\lambda+1}$$

2° $\lambda=1$, $D=0$, $D_x \neq 0 \Rightarrow$ sistem nema rješenja

3° $\lambda=-1$, $D=0$, $D_x \neq 0 \Rightarrow$ sistem nema rješenja

4° $\lambda=0$, $D=D_x=D_y=D_z=0$ iz ovoga ne možemo ništa zaključiti

Za $\lambda=0$ sistem postaje

$$\begin{aligned} x - y &= 1 & (1) \\ y - z &= 0 & (2) \\ x - z &= 1 & (3) \end{aligned}$$

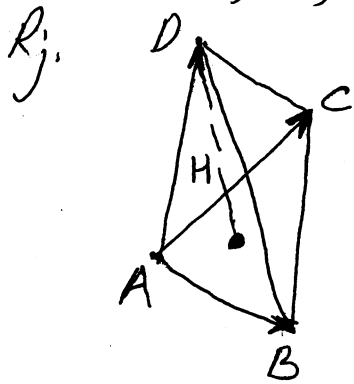
(1): $x - y = 1$

(2)-(3): $-x + y = -1$
 $x = y + 1$

$x - z = 1$
 $-z = -(y+1) + 1$

$-z = -y$
 $z = y$ Sistem ima ∞ mnogo rješenja $(t+1, t, t)$, $t \in \mathbb{R}$

⊕ Izračunati površinu tetraedra ABCD i visinu spuštenu iz vrha D ako je $A(-1, 3, 1)$, $B(5, 3, 8)$, $C(-1, -3, 5)$ i $D(2, 1, -4)$.



Površina tetraedra koje razapinju vektori \vec{AB} , \vec{AC} i \vec{AD} se računa po formuli $P = \frac{1}{6} |(\vec{AB} \times \vec{AC}) \cdot \vec{AD}|$

$$\left. \begin{array}{l} A(-1, 3, 1) \\ B(5, 3, 8) \end{array} \right\} \Rightarrow \vec{AB} = (6, 0, 7)$$

$$\left. \begin{array}{l} A(-1, 3, 1) \\ C(-1, -3, 5) \end{array} \right\} \Rightarrow \vec{AC} = (0, -6, 4)$$

$$\left. \begin{array}{l} A(-1, 3, 1) \\ D(2, 1, -4) \end{array} \right\} \Rightarrow \vec{AD} = (3, -2, -5)$$

$$\begin{aligned} (\vec{AB} \times \vec{AC}) \cdot \vec{AD} &= \begin{vmatrix} 6 & 0 & 7 \\ 0 & -6 & 4 \\ 3 & -2 & -5 \end{vmatrix} = \begin{vmatrix} 0 & 4 & 17 \\ 0 & -6 & 4 \\ 3 & -2 & -5 \end{vmatrix} = 3 \begin{vmatrix} 4 & 17 \\ -6 & 4 \end{vmatrix} \\ &= 3(16 + 102) = 3 \cdot 118 = 354. \end{aligned}$$

$$P = \frac{1}{6} \cdot 354 = 59 \quad \text{tražena površina}$$

Površina tetraedra se može izračunati i pomoću formule

$$P = \frac{1}{3} B \cdot H_0, \quad \text{gdje je } B \text{ površina } \triangle ABC \text{ a } H_0 \text{ visina}$$

spuštena iz vrha D.

$$P_{\triangle ABC} = \frac{1}{2} |\vec{AB} \times \vec{AC}| \quad \vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 0 & 7 \\ 0 & -6 & 4 \end{vmatrix} = (42, -24, -36)$$

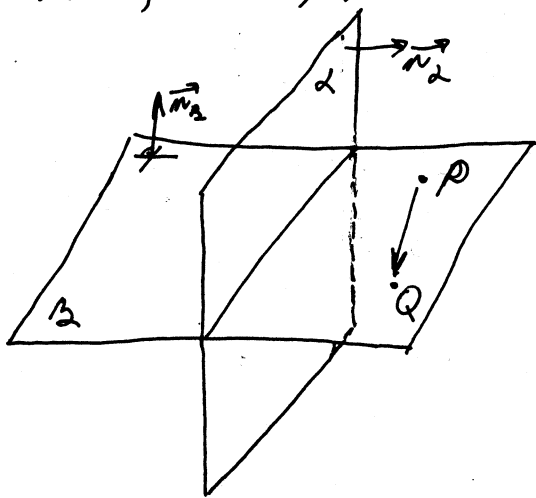
$$\begin{aligned} |\vec{AB} \times \vec{AC}| &= \sqrt{42^2 + 24^2 + 36^2} = \sqrt{6^2 \cdot 7^2 + 6^2 \cdot 4^2 + 6^2 \cdot 6^2} = 6 \sqrt{49 + 16 + 36} \\ &= 6 \sqrt{101} \end{aligned}$$

$$P = \frac{1}{3} B \cdot H_0 \Rightarrow 354 = \frac{1}{3} \cdot 6 \sqrt{101} \cdot H_0 \Rightarrow H_0 = \frac{354}{2\sqrt{101}} = \frac{177}{\sqrt{101}}$$

Visina spuštenu iz vrha D iznosi $\frac{177}{\sqrt{101}}$.

#) Napisati jednačinu ravni koja prolazi kroz tačke $P(1,1,1)$, $Q(0,1,-1)$ i normalna je na ravan $L: x+y+z-1=0$.

Rj.



B: ?

$$\left. \begin{array}{l} \vec{n}_B \perp \vec{PQ} \\ \vec{n}_B \perp \vec{n}_L \end{array} \right\} \Rightarrow \vec{n}_B \parallel \vec{n}_L \times \vec{PQ}$$

$$\Downarrow \\ \exists k \in \mathbb{R}: \vec{n}_B = k(\vec{n}_L \times \vec{PQ})$$

$$P(1,1,1) \\ Q(0,1,-1) \Rightarrow \vec{PQ} = (-1, 0, -2)$$

$$\vec{n}_L = (1, 1, 1)$$

$$\vec{n}_B \times \vec{PQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ -1 & 0 & -2 \end{vmatrix} = \vec{i}(-2-0) - \vec{j}(-2+1) + \vec{k}(0+1) = -2\vec{i} + \vec{j} + \vec{k} \\ = (-2, 1, 1)$$

$$\Rightarrow \vec{n}_B = k(-2, 1, 1) \text{ gdje je } k \text{ neki realan}$$

$$\vec{n}_B = (-2k, k, k) \quad k \neq 0 \text{ (broj)}$$

$$A(x-x_1) + B(y-y_1) + C(z-z_1) = 0 \quad \begin{matrix} x_1 & y_1 & z_1 \\ P(1,1,1) \end{matrix}$$

$$-2k(x-1) + k(y-1) + k(z-1) = 0 \quad | :k$$

$$-2x + 2 + y - 1 + z - 1 = 0$$

$$-2x + y + z = 0 \text{ jednačina tražene ravni}$$

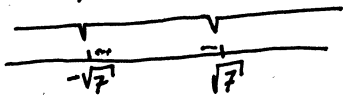
Ispitati f-ju i nacrtati joj grafik $y = \frac{x^3 - 10}{x^2 - 7}$.

f.) DEFINICIONO PODRUČJE

$$x^2 - 7 \neq 0$$

$$x^2 \neq 7$$

$$x_{1,2} \neq \pm\sqrt{7} \approx \pm 2,64$$

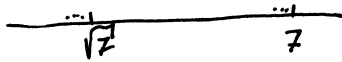


$$D: x \in (-\infty, -\sqrt{7}) \cup (-\sqrt{7}, \sqrt{7}) \cup (\sqrt{7}, +\infty)$$

PARNOST (NEPARNOST), PERIODIČNOST

$$f(-x) = \frac{(-x)^3 - 10}{(-x)^2 - 7} = \frac{-x^3 - 10}{x^2 - 7} \neq \pm f(x)$$

f-ja nije ni parna ni neparna
f-ja nije periodična



NULE, PRESEK SA Y-OSOM, ZNAK F-JE

nule ili presjek sa x-osom

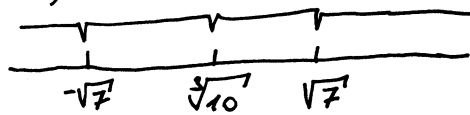
$$y=0 \text{ akko } x^3 - 10 = 0$$

$$x^3 = 10 \Rightarrow x = \sqrt[3]{10} \approx 2,15$$

Nula f-je je $N(\sqrt[3]{10}, 0)$

$$f(0) = \frac{0 - 10}{0 - 7} = \frac{10}{7} \approx 1,42$$

Presjek sa y-osom je $M(0, \frac{10}{7})$.



← prekidi f-je y + nule y

	$(-\infty, -\sqrt{7})$	$(-\sqrt{7}, \sqrt[3]{10})$	$(\sqrt[3]{10}, \sqrt{7})$	$(\sqrt{7}, +\infty)$
$x^3 - 10$	-	-	• +	+
$x^2 - 7$	+	• -	-	• +
Y	-	+	-	+

znak f-je

PONAŠANJE NA KRAJEVIMA INTERVALA DEFINIČANOSTI I ASIMPTOTE

F-ja ima prekid za $x = \pm\sqrt{7}$

$$\lim_{x \rightarrow -\sqrt{7}-0} f(x) = \lim_{x \rightarrow -\sqrt{7}-0} \frac{x^3 - 10}{x^2 - 7} = \frac{-7\sqrt{7} - 0 - 10}{7 + 0 - 7} = -\infty$$

$$\lim_{x \rightarrow -\sqrt{7}+0} f(x) = \lim_{x \rightarrow -\sqrt{7}+0} \frac{x^3 - 10}{x^2 - 7} = \frac{-7\sqrt{7} + 0 - 10}{7 + 0 - 7} = +\infty$$

$$\lim_{x \rightarrow \sqrt{7}-0} f(x) = \lim_{x \rightarrow \sqrt{7}-0} \frac{x^3 - 10}{x^2 - 7} = \frac{7\sqrt{7} - 0 - 10}{7 + 0 - 7} = -\infty$$

$$\lim_{x \rightarrow \sqrt{7}+0} f(x) = \lim_{x \rightarrow \sqrt{7}+0} \frac{x^3 - 10}{x^2 - 7} = \frac{7\sqrt{7} + 0 - 10}{7 + 0 - 7} = +\infty$$

$$\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} \frac{x^3 - 10}{x^2 - 7} \stackrel{\cdot x^2}{=} \lim_{x \rightarrow -\infty} \frac{x - \frac{10}{x^2}}{1 - \frac{7}{x^2}} = \frac{-\infty}{1} = -\infty$$

\Rightarrow f-ja nema H₀A.

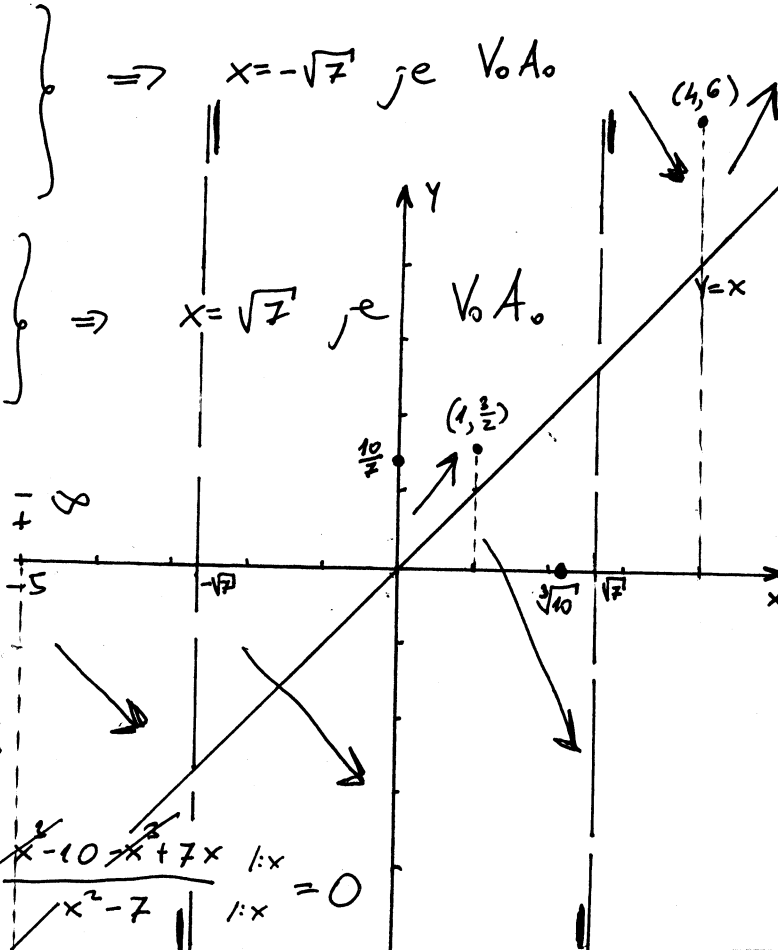
Tražimo K₀A₀ u obliku $y = kx + n$

$$k = \lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{x^3 - 10}{x^2 - 7x} \stackrel{\cdot x^2}{=} \lim_{x \rightarrow \infty} \frac{x - \frac{10}{x^2}}{1 - \frac{7}{x}} = 1$$

$$n = \lim_{x \rightarrow \infty} [f(x) - kx] = \lim_{x \rightarrow \infty} \left(\frac{x^3 - 10}{x^2 - 7} - x \right) = \lim_{x \rightarrow \infty} \frac{x^3 - 10 - x^2 + 7x}{x^2 - 7} \stackrel{\cdot x}{=} \lim_{x \rightarrow \infty} \frac{x^3 - x^2 + 7x - 10}{x^2 - 7} = 0$$

$\Rightarrow x = -\sqrt{7}$ je V₀A₀

$\Rightarrow x = \sqrt{7}$ je V₀A₀



$Y=x$ je K.O.A. Prati je ovog koraka počinjemo skicirati grafik f-je.

RAST I OPADANJE

$$Y' = \left(\frac{x^3 - 10}{x^2 - 7} \right)' = \frac{3x^2(x^2 - 7) - (x^3 - 10)2x}{(x^2 - 7)^2} = \frac{x(3x^3 - 21x(-2x^3) + 20)}{(x^2 - 7)^2} =$$

$$= \frac{x(x^3 - 21x + 20)}{(x^2 - 7)^2} = \frac{x(x-1)(x^2 + x - 20)}{(x^2 - 7)^2} = \frac{x(x-1)(x-4)(x+5)}{(x^2 - 7)^2}$$

$(x^3 - 21x + 20) : (x-1) = x^2 + x - 20$ $Y'=0$ akko $x=0$ ili $x=1$ ili $x=4$ ili $x=-5$

$$\begin{array}{r} x^3 - x^2 \\ - x^2 - 21x + 20 \\ \hline -20x + 20 \\ -20x + 20 \\ \hline = \end{array}$$



↑ prekid y + nule y'

	-100	-4	-1	1/2	2	3	100
x	$(-\infty, -5)$	$(-5, -\sqrt{7})$	$(-\sqrt{7}, 0)$	$(0, 1)$	$(1, \sqrt{7})$	$(\sqrt{7}, 4)$	$(4, +\infty)$
y'	+	-	-	+	-	-	+
Y	↑	↓	↓	↑	↓	↓	↑

MAX MIN MAX MIN rast i opadanje

EKSTREMI F-JE

$$f(-5) = \frac{(-5)^3 - 10}{(-5)^2 - 7} = \frac{-125 - 10}{25 - 7} = \frac{-135}{18} : 3 = \frac{-15}{2} = -7,5$$

$$f(0) = \frac{0^3 - 10}{0^2 - 7} = \frac{-10}{-7} \approx 1,42$$

$$f(1) = \frac{1 - 10}{1 - 7} = \frac{-9}{-6} = \frac{3}{2} = 1,5$$

$$f(4) = \frac{64 - 10}{16 - 7} = \frac{54}{9} = 6$$

Na osnovu tabele rast i opadanja f-ja ima maksimum u tačkama $(-5, -\frac{15}{2})$ i $(1, \frac{3}{2})$ i minimum u tačkama $(0, \frac{10}{7})$ i $(4, 6)$.

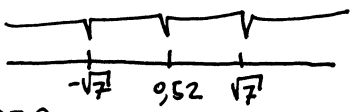
PREVOJNE TAČKE I INTERVALI KONVEKSNOSTI I KONKAVNOSTI

$$Y'' = \left(\frac{x^4 - 21x^2 + 20x}{(x^2 - 7)^2} \right)' =$$

$$= \frac{(4x^3 - 42x + 20)(x^2 - 7)^2 - (x^4 - 21x^2 + 20x)2(x^2 - 7) \cdot 2x}{(x^2 - 7)^4}$$

$$= \frac{4x^5 - 42x^3 + 20x^2 - 28x^3 + 254x - 140 - 4x^5 + 84x^3 + 80x^2}{(x^2 - 7)^3}$$

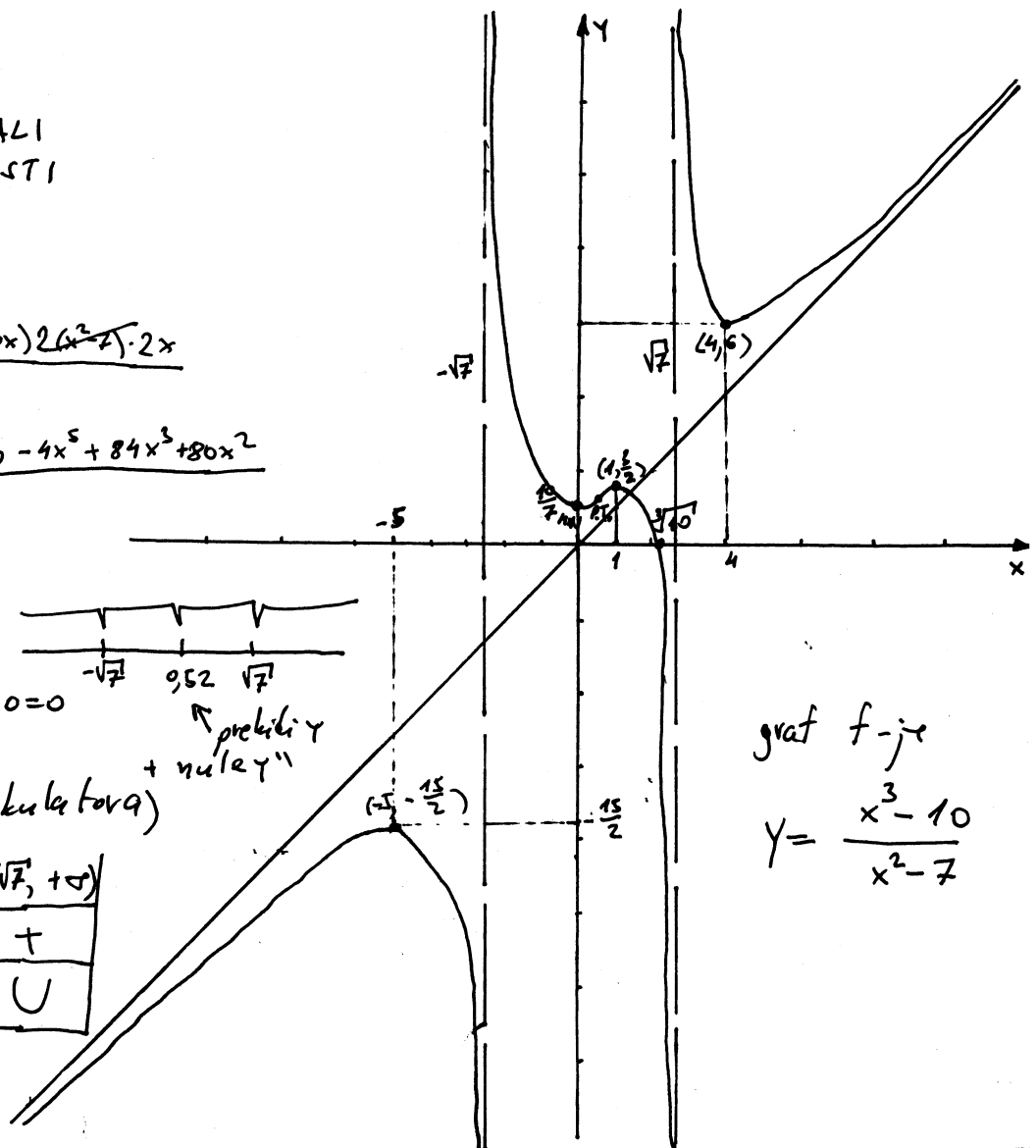
$$= 2 \frac{7x^3 + 147x - 30x^2 - 70}{(x^2 - 7)^3}$$



$Y''=0$ akko $7x^3 + 147x - 30x^2 - 70 = 0$
 $Y''=0$ akko $x = 0,52$
 (nula dobijena uz pomoć kalkulatora)

x	$(-\infty, -\sqrt{7})$	$(-\sqrt{7}, 0,52)$	$(0,52, \sqrt{7})$	$(\sqrt{7}, +\infty)$
y''	-	+	-	+
Y	∩	∪	∩	∪

P.T. konveksnost i konkavnost



graf f-je
 $Y = \frac{x^3 - 10}{x^2 - 7}$