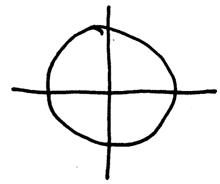
Izračunati $\lim_{x \to -\frac{\pi}{4}} \frac{\sin(4x)\cos(2x)}{\left(x+\frac{\pi}{4}\right)^2}$.

Rj:

$$\lim_{x \to -\frac{\pi}{4}} \frac{\sin(4x)\cdot\cos(2x)}{\left(x+\frac{\pi}{4}\right)^2} \quad \left(=\frac{0}{0}\right)$$



$$\sin(A+B) = \sin A\cos B + \cos A \sin B$$

$$+ \quad \sin(A-B) = \sin A\cos B - \cos A \sin B$$

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$\sin A\cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

$$\sin 4x \cdot \cos 2x = \frac{1}{2}(\sin 6x + \sin 2x)$$

$$\sin 6\cdot\left(-\frac{\pi}{4}\right) = -\sin\frac{3\pi}{2} = -(-1) = 1$$

$$\sin 2\cdot\left(-\frac{\pi}{4}\right) = -\sin\frac{\pi}{2} = -1$$

$$\cos 6\cdot\left(-\frac{\pi}{4}\right) = \cos\frac{3\pi}{2} = 0$$

$$\cos 2\cdot\left(-\frac{\pi}{4}\right) = \cos\frac{\pi}{2} = 0$$

$$\lim_{x \to -\frac{\pi}{4}} \frac{\sin 4x\cdot\cos 2x}{\left(x+\frac{\pi}{4}\right)^2} = \frac{1}{2}\lim_{x \to -\frac{\pi}{4}} \frac{\sin 6x + \sin 2x}{\left(x+\frac{\pi}{4}\right)^2} \left(=\frac{\overset{(1-1)}{0}}{0}\right) \underset{=}{\overset{L.P.}{=}}$$

$$= \frac{1}{2}\lim_{x \to -\frac{\pi}{4}} \frac{\cos(6x)\cdot 6 + \cos(2x)\cdot 2}{2\left(x+\frac{\pi}{4}\right)} \left(=\frac{0+0}{0}\right) \underset{=}{\overset{L.P.}{=}}$$

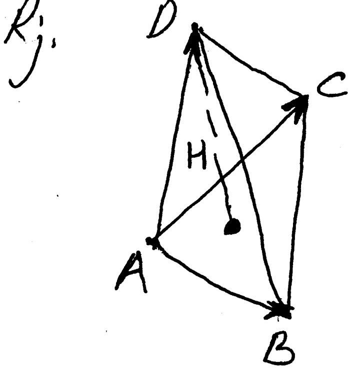
$$= \frac{1}{4}\lim_{x \to -\frac{\pi}{4}} \frac{6\cdot(-6)\sin 6x + 2\cdot(-2)\sin 2x}{1} = \frac{1}{4}\left(-36\cdot 1 - 4\cdot(-1)\right)$$

$$= \frac{1}{4}(4-36) = -\frac{32}{4} = -8$$

traženo rješenje

(#) Izračunati površinu tetraedra $ABCD$ i visinu spuštenu iz vrha D ako je $A(-1,3,1)$, $B(5,3,8)$, $C(-1,-3,5)$ i $D(2,1,-4)$.

Rj.



Površina tetraedra kojeg razapinju vektori $\vec{AB}$, $\vec{AC}$ i $\vec{AD}$ se računa po formuli $P=\frac{1}{6}\left|(\vec{AB}\times\vec{AC})\cdot\vec{AD}\right|$

$$\left.\begin{array}{l} A(-1,3,1) \\ B(5,3,8) \end{array}\right\} \Rightarrow \vec{AB}=(6,0,7)$$

$$\left.\begin{array}{l} A(-1,3,1) \\ C(-1,-3,5) \end{array}\right\} \Rightarrow \vec{AC}=(0,-6,4)$$

$$\left.\begin{array}{l} A(-1,3,1) \\ D(2,1,-4) \end{array}\right\} \Rightarrow \vec{AD}=(3,-2,-5)$$

$$(\vec{AB}\times\vec{AC})\cdot\vec{AD}=\begin{vmatrix} 6 & 0 & 7 \\ 0 & -6 & 4 \\ 3 & -2 & -5 \end{vmatrix} \overset{I_v - III_v\cdot 2}{=\!=\!=} \begin{vmatrix} 0 & 4 & 17 \\ 0 & -6 & 4 \\ 3 & -2 & -5 \end{vmatrix} = 3\begin{vmatrix} 4 & 17 \\ -6 & 4 \end{vmatrix}$$

$$=3(16+102)=3\cdot 118=354.$$

$P=\frac{1}{6}\cdot 354=59$ tražena površina

Površina tetraedra se može izračunati i pomoću formule $P=\frac{1}{3}B\cdot H_D$, gdje je B površina ΔABC a H_D visina spuštena iz vrha D.

$$P_{\Delta ABC}=\frac{1}{2}\left|\vec{AB}\times\vec{AC}\right| \qquad \vec{AB}\times\vec{AC}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & 0 & 7 \\ 0 & -6 & 4 \end{vmatrix}=(42,-24,-36)$$

$$\left|\vec{AB}\times\vec{AC}\right|=\sqrt{42^2+24^2+36^2}=\sqrt{6^2\cdot 7^2+6^2\cdot 4^2+6^2\cdot 6^2}=6\sqrt{49+16+36}$$
$$=6\sqrt{101}$$

$$P=\frac{1}{3}B\cdot H_D \Rightarrow 354=\frac{1}{\cancel{3}_1}\cdot\cancel{6}^{\,2}\sqrt{101}\cdot H_D \Rightarrow H_D=\frac{354}{2\sqrt{101}}=\frac{177}{\sqrt{101}}$$

Visina spuštena iz vrha D iznosi $\frac{177}{\sqrt{101}}$.

Ispitati f-ju i nacrtati joj grafik $y = \dfrac{x^3-10}{x^2-7}$.

R:

DEFINICIONO PODRUČJE

$x^2-7 \neq 0$

$x^2 \neq 7$

$x_{1,2} \neq \pm\sqrt{7} \approx \pm 2{,}64$

$-\sqrt{7} \qquad \sqrt{7}$

$D: x \in (-\infty, -\sqrt{7}) \cup (-\sqrt{7}, \sqrt{7}) \cup (\sqrt{7}, +\infty)$

PARNOST (NEPARNOST), PERIODIČNOST

$$f(-x) = \frac{(-x)^3-10}{(-x)^2-7} = \frac{-x^3-10}{x^2-7} \neq \pm f(x)$$

f-ja nije ni parna ni neparna

f-ja nije periodična

$\sqrt{7} \qquad 7$

NULE, PRESJEK SA Y-OSOM, ZNAK F-JE

nule ili presjek sa x-osom

$y=0$ akko $x^3-10=0$

$x^3 = 10 \Rightarrow x = \sqrt[3]{10} \approx 2{,}15$

Nula f-je je $N(\sqrt[3]{10}, 0)$

$$f(0) = \frac{0-10}{0-7} = \frac{10}{7} \approx 1{,}42$$

Presjek sa y-osom je $M\left(0, \frac{10}{7}\right)$.

$-\sqrt{7} \qquad \sqrt[3]{10} \qquad \sqrt{7}$ ← prekidi f-je y + nule y

	$(-\infty, -\sqrt{7})$	$(-\sqrt{7}, \sqrt[3]{10})$	$(\sqrt[3]{10}, \sqrt{7})$	$(\sqrt{7}, +\infty)$
x^3-10	−	−	+	+
x^2-7	+	−	−	+
y	−	+	−	+

znak f-je

PONAŠANJE NA KRAJEVIMA INTERVALA DEFINISANOSTI I ASIMPTOTE

F-ja ima prekid za $x = \pm\sqrt{7}$

$$\lim_{x\to-\sqrt{7}-0} f(x) = \lim_{x\to-\sqrt{7}-0} \frac{x^3-10}{x^2-7} = \frac{-7\sqrt{7}-0-10}{7+0-7} = -\infty$$

$$\lim_{x\to-\sqrt{7}+0} f(x) = \lim_{x\to-\sqrt{7}+0} \frac{x^3-10}{x^2-7} = \frac{-7\sqrt{7}+0-10}{7-0-7} = +\infty$$

$\Rightarrow x = -\sqrt{7}$ je V.A.

$$\lim_{x\to\sqrt{7}-0} f(x) = \lim_{x\to\sqrt{7}-0} \frac{x^3-10}{x^2-7} = \frac{7\sqrt{7}-0-10}{7-0-7} = -\infty$$

$$\lim_{x\to\sqrt{7}+0} f(x) = \lim_{x\to\sqrt{7}+0} \frac{x^3-10}{x^2-7} = \frac{7\sqrt{7}+0-10}{7+0-7} = +\infty$$

$\Rightarrow x = \sqrt{7}$ je V.A.

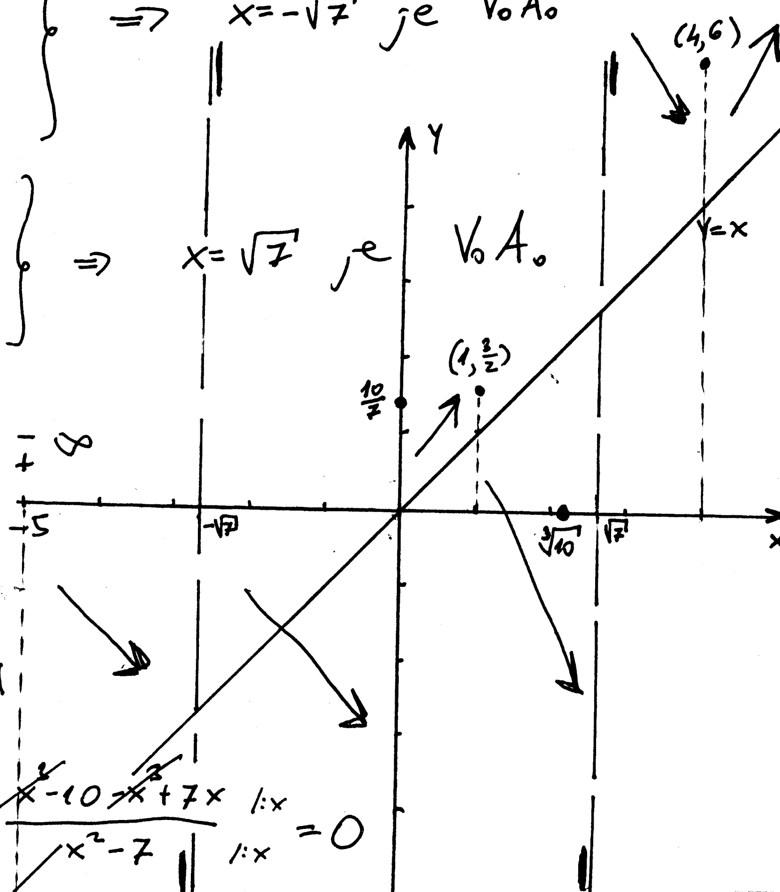
$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{x^3-10 \;/:x^2}{x^2-7 \;/:x^2} = \lim_{x\to\pm\infty} \frac{x-\cancel{\frac{10}{x^2}}\to 0}{1-\cancel{\frac{7}{x^2}}\to 0} = \pm\infty$$

$\Rightarrow$ f-ja nema H.A.

Tražimo K.A. u obliku $y = kx+n$

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^3-10 \;/:x^3}{x^3-7x \;/:x^3} = 1$$

$$n = \lim_{x\to\infty} [f(x)-kx] = \lim_{x\to\infty}\left(\frac{x^3-10}{x^2-7} - x\right) = \lim_{x\to\infty} \frac{\cancel{x^3}-10-\cancel{x^3}+7x \;/:x}{x^2-7 \;/:x} = 0$$



$y = x$ je K.A. Poslije ovog koraka počinjemo skicirati grafik f-je.

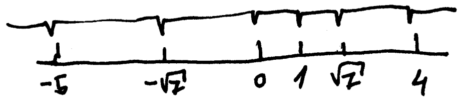
RAST I OPADANJE

$$y' = \left(\frac{x^3-10}{x^2-7}\right)' = \frac{3x^2(x^2-7)-(x^3-10)2x}{(x^2-7)^2} = \frac{x(3x^3-21x-2x^3+20)}{(x^2-7)^2} =$$

$$= \frac{x(x^3-21x+20)}{(x^2-7)^2} = \frac{x(x-1)(x^2+x-20)}{(x^2-7)^2} = \frac{x(x-1)(x-4)(x+5)}{(x^2-7)^2}$$

$$(x^3-21x+20):(x-1) = x^2+x-20$$
$$\underline{-\;x^3-x^2}$$
$$x^2-21x+20$$
$$\underline{-\;x^2-x}$$
$$-20x+20$$
$$\underline{-\;-20x+20}$$
$$//\quad //$$

$y' = 0$ akko $x=0$ ili $x=1$ ili $x=4$ ili $x=-5$



	-100	-4	-1	$\frac{1}{2}$	2	3	100
x	$(-\infty,-5)$	$(-5,-\sqrt{7})$	$(-\sqrt{7},0)$	$(0,1)$	$(1,\sqrt{7})$	$(\sqrt{7},4)$	$(4,+\infty)$
y'	+	−	−	+	−	−	+
y	↗	↘	↘	↗	↘	↘	↗
	MAX			MIN	MAX		MIN

rast i opadanje

EKSTREMI F-JE

$$f(-5) = \frac{(-5)^3-10}{(-5)^2-7} = \frac{-125-10}{25-7} = \frac{-135}{18}\,\frac{:9}{:9} = \frac{-15}{2} = -7,5$$

$$f(0) = \frac{0^3-10}{0^2-7} = \frac{10}{7} \approx 1,42$$

$$f(1) = \frac{1-10}{1-7} = \frac{-9}{-6} = \frac{3}{2} = 1,5$$

$$f(4) = \frac{64-10}{16-7} = \frac{54}{9} = 6$$

Na osnovu tabele rasta i opadanja f-ja ima maksimum u tačkama $(-5,-\frac{15}{2})$ i $(1,\frac{3}{2})$ i minimum u tačkama $(0,\frac{10}{7})$ i $(4,6)$.

PREVOJNE TAČKE I INTERVALI KONVEKSNOSTI I KONKAVNOSTI

$$y'' = \left(\frac{x^4-21x^2+20x}{(x^2-7)^2}\right)' =$$

$$= \frac{(4x^3-42x+20)(x^2-7)^{\cancel{2}\,1}-(x^4-21x^2+20x)2\cancel{(x^2-7)}\cdot 2x}{(x^2-7)^{\cancel{4}\,3}}$$

$$= \frac{4x^5-42x^3+20x^2-28x^3+294x-140-4x^5+84x^3+80x^2}{(x^2-7)^3}$$

$$= 2\,\frac{7x^3+147x-30x^2-70}{(x^2-7)^3}.$$

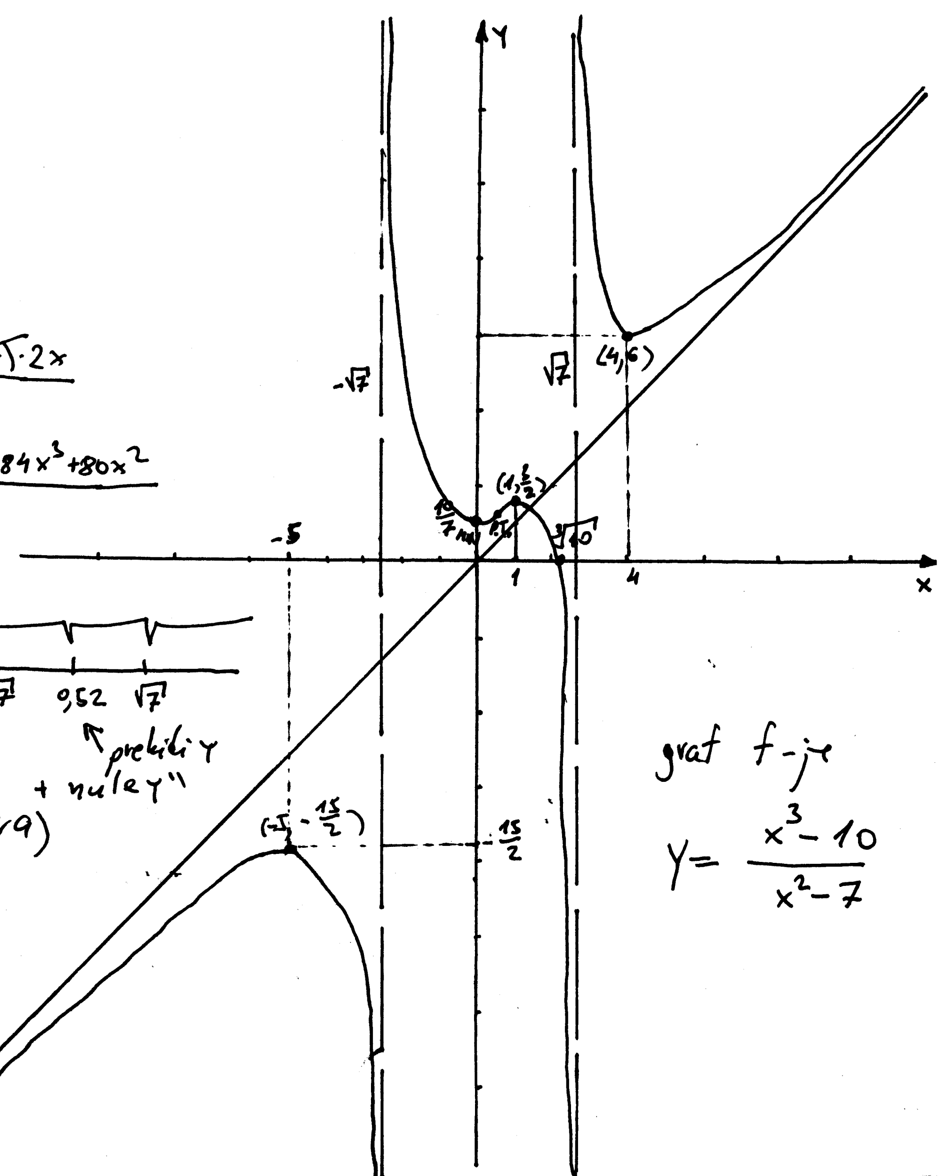
$y'' = 0$ akko $7x^3+147x-30x^2-70=0$

$y'' = 0$ akko $x = 0,52$

(nula dobijena uz pomoć kalkulatora)

x	$(-\infty,-\sqrt{7})$	$(-\sqrt{7},0,52)$	$(0,52,\sqrt{7})$	$(\sqrt{7},+\infty)$
y''	−	+	−	+
y	∩	∪	∩	∪
			P.T.	

konveksnost i konkavnost



graf f-je

$$y = \frac{x^3-10}{x^2-7}$$

(#) Izračunati integral $I=\int \frac{\sin x\cdot\cos x}{e^x}\,dx$.

Rj.

$$I=\int\frac{\sin x\cdot\cos x}{e^x}\,dx=\frac{1}{2}\int\frac{2\sin x\cos x}{e^x}\,dx=\frac{1}{2}\int\frac{\sin 2x}{e^x}\,dx=$$

$$=\frac{1}{2}\int e^{-x}\sin 2x\,dx=\begin{vmatrix} u=e^{-x} & dv=\sin 2x\,dx \\ du=-e^{-x}dx & v=-\frac{1}{2}\cos 2x\end{vmatrix}=$$

$$=\frac{1}{2}\left(-\frac{1}{2}e^{-x}\cos 2x-\frac{1}{2}\int e^{-x}\cos 2x\,dx\right)=$$

$$=-\frac{1}{4}e^{-x}\cos 2x-\frac{1}{4}\int e^{-x}\cos 2x\,dx$$

$$\int e^{-x}\cos 2x\,dx=\begin{vmatrix} u=e^{-x} & dv=\cos 2x\,dx \\ du=-e^{-x}dx & v=\frac{1}{2}\sin 2x\end{vmatrix}=\frac{1}{2}e^{-x}\sin 2x+\frac{1}{2}\int e^{-x}\sin 2x\,dx$$

Dobili smo

$$\underline{\frac{1}{2}\int e^{-x}\sin 2x\,dx}=-\frac{1}{4}e^{-x}\cos 2x-\frac{1}{8}e^{-x}\sin 2x\underline{-\frac{1}{8}\int e^{-x}\sin 2x\,dx}$$

$$\frac{5}{8}\int e^{-x}\sin 2x\,dx=-\frac{1}{4}e^{-x}\cos 2x-\frac{1}{8}e^{-x}\sin 2x \quad /\cdot\frac{8}{10}$$

$$\frac{1}{2}\int e^{-x}\sin 2x\,dx=-\frac{2}{10}e^{-x}\cos 2x-\frac{1}{10}e^{-x}\sin 2x$$

Kako je $I=\int\frac{\sin x\cdot\cos x}{e^x}\,dx=\frac{1}{2}\int e^{-x}\sin 2x\,dx$ to je

$$\int\frac{\sin x\cdot\cos x}{e^x}\,dx=-\frac{1}{10}e^{-x}\left(2\cos 2x+\sin 2x\right)$$