Univerzitet u Zenici
Mašinski fakultet
Odsjek: Opšte mašinstvo
Zenica, 16.09.2010.

Pismeni ispit iz predmeta Matematika 1

1. Kompleksan broj $z = -8 - 8i\sqrt{3}$ pretvoriti u trigonometriski oblik. Poslije toga izračunati $z \cdot (cos\frac{2\pi}{3} + i\, sin\frac{2\pi}{3})$.

2. Riješiti sistem jednačina

$$\begin{aligned} 3x + 2y + 5z &= -3 \\ 6x - 8y - 9z &= 1 \\ 11x + 2y - 7z &= 9 \quad . \end{aligned}$$

3. Date su tačke $A(1,1,2)$, $B(-3,0,2)$, $C(4,-1,4)$, $D(0,1,-1)$ i vektori $\vec{a} = \overrightarrow{AB}$, $\vec{b} = \overrightarrow{BC}$, $\vec{c} = \overrightarrow{CD}$. Izračunati:
a) $(\vec{a} - \vec{c}) \times (2\vec{b} + \vec{c})$;
b) ugao između vektora $\vec{a}$ i $\vec{b}$.

4. Ispitati i grafički predstaviti funkciju $y = \dfrac{x}{(-x-1)^2}$.

(Rješenja su skinuta sa stranice \pf.unze.ba\nabokov
Za sve uočene greške pisati na **infoarrt@gmail.com**)

(#) Kompleksan broj $z=-8-8i\sqrt{3}$ pretvoriti u trigonometrijski oblik. Poslije toga izračunati $z\cdot(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})$.

Rj.

$z=-8-8i\sqrt{3}$

$|z|=\sqrt{64+3\cdot 64}$

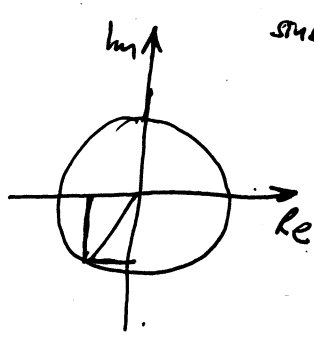
$|z|=\sqrt{4\cdot 64}$

$|z|=2\cdot 8=16$

$\cos\varphi=\frac{a}{|z|}=\frac{-8}{16}=-\frac{1}{2}$

$\sin\varphi=\frac{b}{|z|}=\frac{-8\sqrt{3}}{16}=-\frac{\sqrt{3}}{2}$

$\varphi=\frac{4\pi}{3}=240°$



$z=|z|(\cos\varphi+i\sin\varphi)=16(\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3})$

$z_1=|z_1|(\cos\varphi_1+i\sin\varphi_1)$
$z_2=|z_2|(\cos\varphi_2+i\sin\varphi_2)$
$\Rightarrow z_1\cdot z_2=|z_1||z_2|(\cos(\varphi_1+\varphi_2)+i\sin(\varphi_1+\varphi_2))$

$z\cdot(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})=16(\cos(\frac{4\pi}{3}+\frac{2\pi}{3})+i\sin(\frac{4\pi}{3}+\frac{2\pi}{3}))=16(\cos 2\pi+i\sin 2\pi)$

$z(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3})=16\cdot(1+0)=16$

(#) Date su tačke $A(1,1,2)$, $B(-3,0,2)$, $C(4,-1,4)$, $D(0,1,-1)$, vektori $\vec{a}=\overrightarrow{AB}$, $\vec{b}=\overrightarrow{BC}$, $\vec{c}=\overrightarrow{CD}$. Izračunati

a) $(\vec{a}-\vec{c})\times(2\vec{b}+\vec{c})$

b) ugao između vektora $\vec{a}$ i $\vec{b}$.

Rj.

$A(1,1,2)$, $B(-3,0,2)$ $\Rightarrow \overrightarrow{AB}=(-4,-1,0)$

$B(-3,0,2)$, $C(4,-1,4)$ $\Rightarrow \overrightarrow{BC}=(7,-1,2)$

$C(4,-1,4)$, $D(0,1,-1)$ $\Rightarrow \overrightarrow{CD}=(-4,2,-5)$

a) $\vec{a}=(-4,-1,0)$

$\vec{b}=(7,-1,2)$

$\vec{c}=(-4,2,-5)$

$\vec{a}-\vec{c}=(0,-3,5)$

$2\vec{b}=(14,-2,4)$

$\vec{c}=(-4,2,-5)$ $\Rightarrow 2\vec{b}+\vec{c}=(10,0,-1)$

$$(\vec{a}-\vec{c})\times(2\vec{b}+\vec{c})=\begin{vmatrix}\vec{i}&\vec{j}&\vec{k}\\0&-3&5\\10&0&-1\end{vmatrix}=3\vec{i}+50\vec{j}-30\vec{k}=(3,50,-30)$$

$$(\vec{a}-\vec{c})\times(2\vec{b}+\vec{c})=(3,50,-30)$$

b) $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\sphericalangle(\vec{a},\vec{b})$

$\vec{a}\cdot\vec{b}=(-4,-1,0)(7,-1,2)=-28+1+0=-27$

$|\vec{a}|=\sqrt{16+1+0}=\sqrt{17}$

$|\vec{b}|=\sqrt{49+1+4}=\sqrt{54}=\sqrt{18\cdot 3}=\sqrt{2\cdot 9\cdot 3}=3\sqrt{6}$

$$\cos\sphericalangle(\vec{a},\vec{b})=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}=\frac{-27}{3\sqrt{6}\cdot\sqrt{17}}=\frac{-27}{3\sqrt{102}}$$

$$\sphericalangle(\vec{a},\vec{b})=\operatorname{arc}\cos\frac{-27}{3\sqrt{102}}$$

(#) Riješiti sistem jednačina

$$3x + 2y + 5z = -3$$
$$6x - 8y - 9z = 1$$
$$11x + 2y - 7z = 9$$

Rj.

$3x + 2y + 5z = -3 \quad /\cdot 4$

$6x - 8y - 9z = 1$

$11x + 2y - 7z = 9 \quad /\cdot 4$

$12x + 8y + 20z = -12 \quad (I)$

$6x - 8y - 9z = 1 \quad (II)$

$44x + 8y - 28z = 36 \quad (III)$

(I)+(II): $18x + 11z = -11 \quad /\cdot 37$

(III)+(II): $50x - 37z = 37 \quad /\cdot 11$

$666x + 407z = -407$

$+ \quad 550x - 407z = 407$

$1\,216x = 0$

$x = 0$

$x=0$:

$2y + 5z = -3 \quad (a)$

$-8y - 9z = 1 \quad (b)$

$2y - 7z = 9 \quad (c)$

(a)−(c): $12z = -12$

$z = -1$

$2y - 5 = -3$

$2y = 5 - 3$

$2y = 2$

$y = 1$

Rješenje sistema je $x=0,\ y=1,\ z=-1$.

II način:

$D = 608, \quad D_x = 0, \quad D_y = 608, \quad D_z = -608$

⋮

(#) Ispitati i grafički predstaviti f-ju $y = \frac{x}{(-x-1)^2}$.

Dj. definiciono područje

$-x-1 \neq 0$

$x \neq -1$

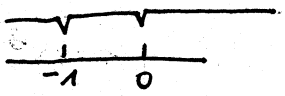
$D: x \in \mathbb{R} \setminus \{-1\}$

$x \in (-\infty, -1) \cup (-1, +\infty)$

nule, presjek sa Y-osom, znak f-je

$Y = 0$ akko $x = 0$

$(0,0)$ je nula f-je i presjek sa Y-osom



x	$(-\infty,-1)$	$(-1,0)$	$(0,+\infty)$
Y	$-$	$-$	$+$

znak f-je

- parnost (neparnost), periodičnost

D nije simetrično $\Rightarrow$

f-ja nije ni parna ni neparna

f-ja nije periodična

$(-x-1)^2 = \big((-1)(x+1)\big)^2$

$= (x+1)^2 \geq 0 \ \forall x$

ponašanje na krajevima intervala definisanosti i asimptote

za $x = -1$ f-ja ima prekid

$$\lim_{x \to -1-0} f(x) = \lim_{x \uparrow -1} \frac{x}{(-x-1)^2} = \frac{-1-0}{(-(-1-0)-1)^2} = \frac{-1-0}{+0} = -\infty$$

$$\lim_{x \downarrow -1} f(x) = \lim_{x \to -1+0} \frac{x}{(-x-1)^2} = \frac{-1+0}{+0} = -\infty$$

$\Rightarrow x = -1$ je $V_0 A_0$

$$Y = \frac{x}{(-x-1)^2} = \frac{x}{(x+1)^2}$$

$$\lim_{\substack{x \to -\infty \\ +}} f(x) = \lim_{x \to -\infty} \frac{x}{(x+1)^2} = \lim_{x \to -\infty} \frac{x \ \ /:x}{x^2+2x+1 \ /:x} = 0 \Rightarrow Y = 0 \text{ je } H_0 A_0$$

f-ja nema kose asimptote

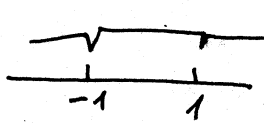
pa prije ovog koraka počinjemo skicirati grafik f-je

rast i opadanje

$$Y' = \left(\frac{x}{(x+1)^2}\right)' = \frac{1 \cdot (x+1)^2 - x \cdot 2(x+1) \cdot 1}{(x+1)^4 \ 3}$$

$$Y' = \frac{x+1-2x}{(x+1)^3} = \frac{1-x}{(x+1)^3}$$

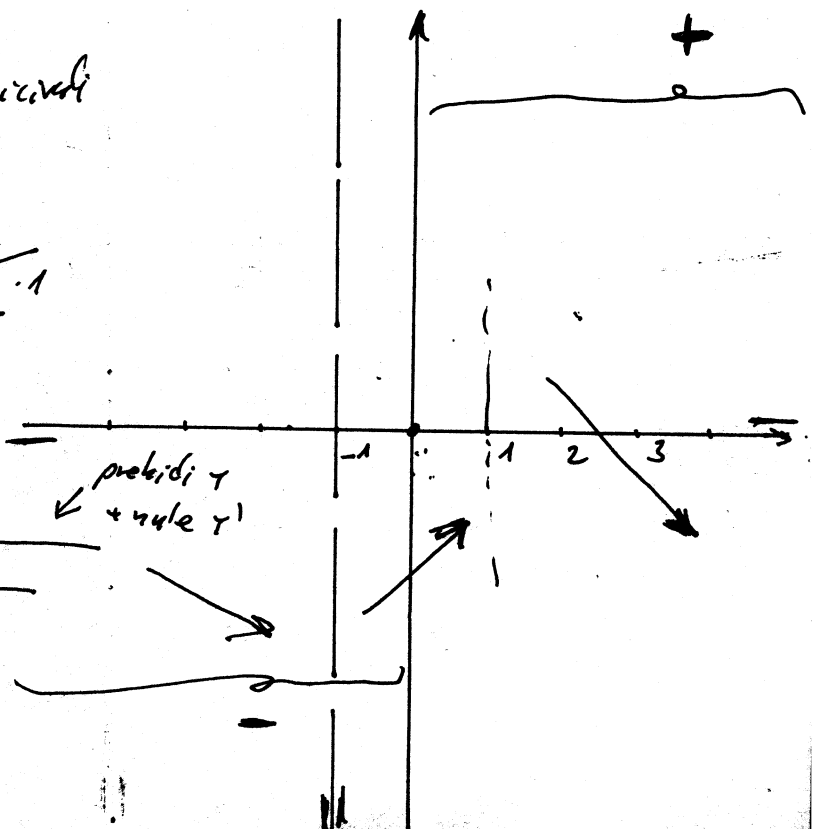
$Y' = 0$ akko $x = 1$



x	$(-\infty,-1)$	$(-1,1)$	$(1,+\infty)$
Y'	$-$	$+$	$-$
Y	$\searrow$	$\nearrow$	$\searrow$

max

rast i opadanje



ekstremi f-je

Na osnovu tabele raste i opadanja, f-ja ima maksimum za $x=1$

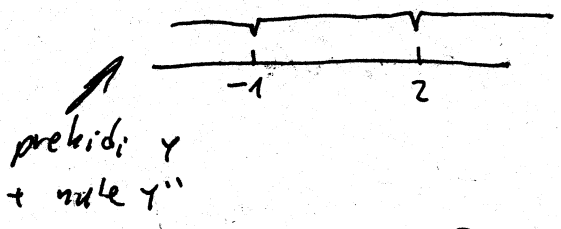
$f(1)=\frac{1}{(1+1)^2}=\frac{1}{4}$ $\quad M(1,\frac{1}{4})$ je maksimum f-je

prevojne tačke i intervali konveksnosti i konkavnosti

$$y''=\left(\frac{1-x}{(x+1)^3}\right)'=\frac{(-1)(x+1)^{1}-(1-x)3(x+1)^{2}}{(x+1)^{6}}=\frac{-x-1-3+3x}{(x+1)^4}=\frac{2x-4}{(x+1)^4}$$

$y''=\frac{2x-4}{(x+1)^4}$ $\quad y''=0$ akko $2x-4=0$

$x=2$



x	$(-\infty,-1)$	$(-1,2)$	$(2,+\infty)$
y''	$-$	$-$	$+$
y	$\cap$	$\cap$	$\cup$

konveks. i konkav.

P.T.

$f(2)=\frac{2}{3^2}=\frac{2}{9}$ $\quad P(2,\frac{2}{9})$ je prevojna tačka