(#) Izračunati x ako u binomnom razvoju $\left(\frac{\sqrt{2^x}}{\sqrt[16]{8}}+\frac{\sqrt[16]{32}}{\sqrt{2^x}}\right)^8$ dobijemo 56 kad oduzmemo šesti od četvrtog člana.

Rj.

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k \qquad 8=2\cdot 4=2^3, \quad 32=2\cdot 16=2\cdot 4^2=2^5$$

$$\left(\frac{\sqrt{2^x}}{\sqrt[16]{8}}+\frac{\sqrt[16]{32}}{\sqrt{2^x}}\right)^8=\left(2^{\frac{x}{2}}\cdot 8^{-\frac{1}{16}}+32^{\frac{1}{16}}\cdot 2^{-\frac{x}{2}}\right)^8=\left(2^{\frac{x}{2}}\cdot 2^{-\frac{3}{16}}+2^{\frac{5}{16}}\cdot 2^{-\frac{x}{2}}\right)^8=$$

$$=\left(2^{\frac{8x-3}{16}}+2^{\frac{5-8x}{16}}\right)^8=\sum_{k=0}^{8}\binom{8}{k}\left(2^{\frac{8x-3}{16}}\right)^{8-k}\left(2^{\frac{5-8x}{16}}\right)^k=$$

$$=\sum_{k=0}^{8}\binom{8}{k}2^{\frac{(8x-3)(8-k)}{16}}\cdot 2^{\frac{(5-8x)k}{16}}=\sum_{k=0}^{8}\binom{8}{k}2^{\frac{64x-24-8kx+3k+5k-8kx}{16}}$$

$$=\sum_{k=0}^{8}\binom{8}{k}2^{\frac{64x-16kx+8k-24}{16}\,/:8}=\sum_{k=0}^{8}\binom{8}{k}2^{\frac{8x-2kx+k-3}{2}}$$

$$=\sum_{k=0}^{8}\binom{8}{k}2^{4x-kx+\frac{k-3}{2}}$$

Četvrti član dobijemo za $k=3$: $\binom{8}{3}2^x=\frac{8\cdot 7\cdot \cancel{6}}{1\cdot 2\cdot \cancel{3}}2^x=56\cdot 2^x$ ...(1)

Šesti član dobijemo za $k=5$: $\binom{8}{5}2^{-x+1}=\binom{8}{3}2^{1-x}=56\cdot 2^{1-x}$ ...(2)

$(1)-(2)=56$

$56\cdot 2^x-56\cdot 2^{1-x}=56 \quad /:56$

$2^x-2\cdot 2^{-x}=1 \quad /\cdot 2^x$

$2^{2x}-2^x-2=1$

$2^x=t$

$t^2-t-2=0$

$D=1+8=9$

$t_{1,2}=\frac{1\pm 3}{2}$ $\quad t_1=-1$ negativno otpada

$t_2=2$

$2^x=2$

$2^x=2^1$

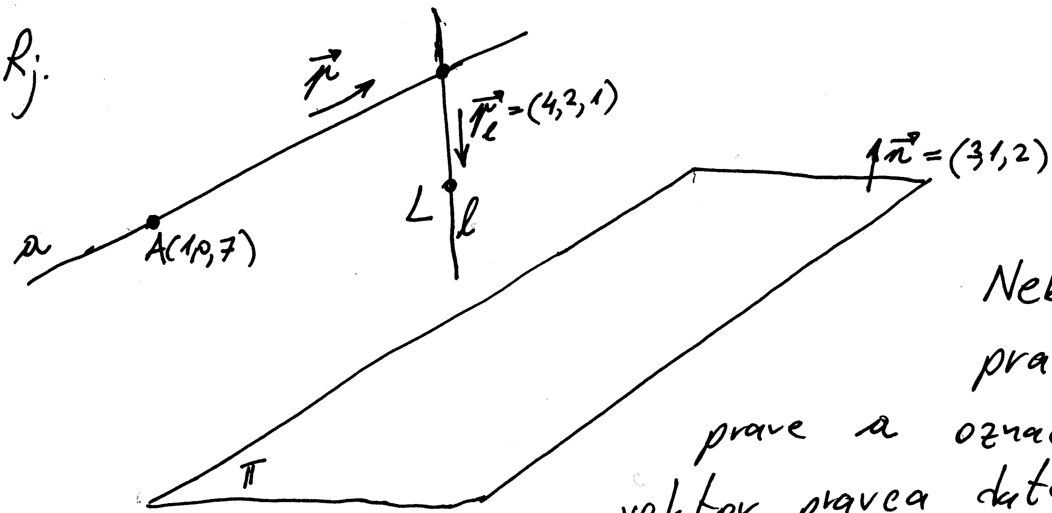
$x=1$

Za $x=1$ u binomnom razvoju dobijemo 56 kada oduzmemo šesti od četvrtog člana.

(#) Kroz tačku $A(1,0,7)$ povući pravu koja je paralelna ravni $\pi: 3x+y+2z+15=0$ i koja siječe pravu

$$l: \frac{x-1}{4} = \frac{y+3}{2} = \frac{z}{1}.$$

Rj.



Neka je a tražena prava. Vektor pravca prave a označimo sa $\vec{p}$, a vektor pravca date prave označimo sa $\vec{p}_l$. (l je data prava)

Pronađimo vektor pravca prave a ($\vec{p}=(l,m,n)$).
Kako su prava a i ravan π paralelne to je $\vec{n} \perp \vec{p}$, pa

$$\vec{n}\cdot\vec{p}=0 \implies (3,1,2)\cdot(l,m,n)=0$$

$$3l+m+2n=0 \quad \ldots(1)$$

Iz jednačine prave l vidimo da je $L(1,-3,0)$ tačka na pravoj l.

$$\begin{matrix} A(1,0,7) \\ L(1,-3,0) \end{matrix} \implies \vec{AL}=(0,-3,-7)$$

Kako se prava a i prava l sijeku to su one u istoj ravni pa imamo da $(\vec{AL}\times\vec{p})\cdot\vec{p}_l=0$ tj.

$$\begin{vmatrix} 0 & -3 & -7 \\ l & m & n \\ 4 & 2 & 1 \end{vmatrix}=0.$$

$$\begin{vmatrix} 0 & -3 & -7 \\ l & m & n \\ 4 & 2 & 1 \end{vmatrix} = (-l)\begin{vmatrix} -3 & -7 \\ 2 & 1 \end{vmatrix} + 4\cdot\begin{vmatrix} -3 & -7 \\ m & n \end{vmatrix} =$$

$$= (-l)(-3+14)+4(-3n+7m) = -11l-12n+28m$$

$$-11l-12n+28m=0 \quad \ldots(2)$$

(1) i (2)

$$\begin{aligned} 3l+m+2n&=0 \quad /\cdot 6 \\ -11l+28m-12n&=0 \end{aligned}$$

$$\begin{aligned} 18l+6m+12n&=0 \\ + \quad -11l+28m-12n&=0 \end{aligned}$$

$$7l+34m=0$$

$$l=-\frac{34}{7}m$$

$$3l+m+2n=0$$

$$-\frac{102}{7}m+m+2n=0$$

$$-\frac{95}{7}m+2n=0$$

$$2n=\frac{95}{7}m \implies n=\frac{95}{14}m$$

Prema tome za vektor pravca prave a imamo

$$\vec{p} = \left(-\frac{34}{7}m, m, \frac{95}{14}m\right),$$ gdje je $m \in \mathbb{R}$.

Jednačina prave kroz tačku (x_1, y_1, z_1) je $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$.

Kako je $A(1,0,7)$ data tačka imamo

$$\frac{x-1}{-\frac{34}{7}} = \frac{y}{1} = \frac{z-7}{\frac{95}{14}}$$ ← jednačina tražene prave

Ako jednačinu pomnožimo sa 14 u ljepšem obliku imamo

$$\frac{x-1}{-68} = \frac{y}{14} = \frac{z-7}{95}$$

Ispitati f-ju i nacrtati joj grafik $Y = \dfrac{2e^{x}+e^{-x}}{e^{x}-2e^{-x}}$.

Rj. Napišimo f-ju u ljepšem obliku

$$Y = \frac{2e^{x}+e^{-x}}{e^{x}-2e^{-x}} \begin{matrix} /\cdot e^{x} \\ /\cdot e^{x} \end{matrix} = \frac{2e^{2x}+1}{e^{2x}-2}$$

definiciono područje

$$\mathcal{D}: x \in \mathbb{R} \setminus \left\{ \frac{\ln 2}{2} \right\}$$

$$e^{2x}-2 \neq 0$$

$$e^{2x} \neq 2 \quad /\ln$$

$$2x \neq \ln 2$$

nule, presjek sa Y-osom, znak

$Y=0$ akko $2e^{2x}+1=0$

$$2e^{2x}=-1$$

$$e^{2x}=-\frac{1}{2} \ \#$$

f-ja nema nulu

$$x=0 \Rightarrow Y=\frac{2+1}{1-2}=-3$$

$(0,-3)$ je presjek sa Y-osom

$$2e^{2x}+1>0 \quad \forall x \in \mathbb{R}$$

$$e^{2x}-2>0$$

$$e^{2x}>2 \quad /\ln$$

$$2x>\ln 2$$

$$x>\frac{\ln 2}{2} \approx 0{,}34$$

$\frac{\ln 2}{2}$ nule Y + polovi Y

parnost (neparnost), periodičnost

$$f(-x)=\frac{2e^{-2x}+1}{e^{-2x}-2} \neq \begin{cases} f(x) \\ -f(x) \end{cases}$$

f-ja nije ni parna ni neparna

f-ja nije periodična

x	$\left(-\infty, \frac{\ln 2}{2}\right)$	$\left(\frac{\ln 2}{2}, +\infty\right)$
Y	$-$	$+$

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

VERTIKALNA ASIMPTOTA

$$\lim_{x \to \frac{\ln 2}{2}+0} f(x) = \lim_{x \to \frac{\ln 2}{2}+0} \frac{2e^{2x}+1}{e^{2x}-2} = \frac{2e^{\ln 2+0}+1}{e^{\ln 2+0}-2} = \frac{4+0+1}{2+0-2} = +\infty$$

$$\lim_{x \to \frac{\ln 2}{2}-0} f(x) = \lim_{x \to \frac{\ln 2}{2}-0} \frac{2e^{2x}+1}{e^{2x}-2} = \frac{2e^{\ln 2-0}+1}{e^{\ln 2-0}-2} = \frac{4-0+1}{2-0-2} = -\infty$$

$\Rightarrow x=\frac{\ln 2}{2}$ je V.A.

HORIZONTALNA ASIMPTOTA

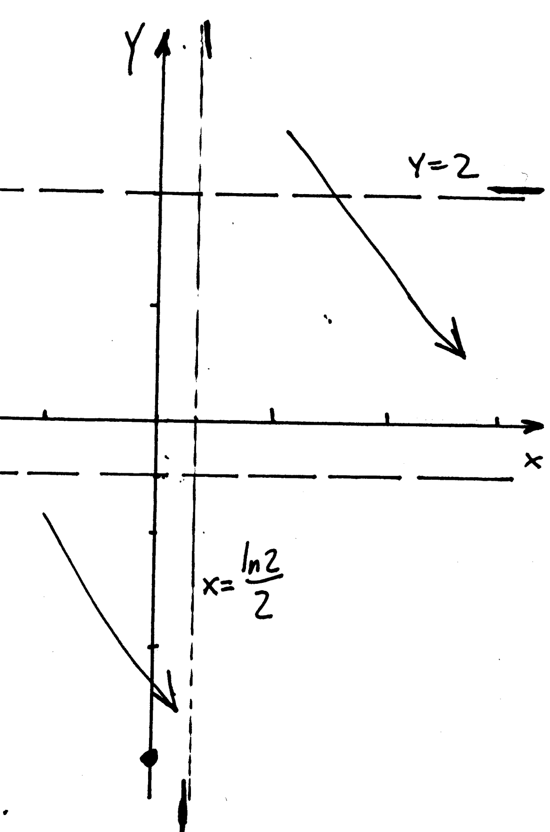
$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{2e^{2x}+1}{e^{2x}-2} = \frac{2e^{-\infty}+1}{e^{-\infty}-2} = -\frac{1}{2}$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{2e^{2x}+1}{e^{2x}-2} \left(\frac{\infty}{\infty}\right) \overset{L.P.}{=} \lim_{x \to +\infty} \frac{2e^{2x}\cdot 2}{e^{2x}\cdot 2} = 2$$

$Y=-\frac{1}{2}$ je V.A. kad $x \to +\infty$

$Y=2$ je V.A. kad $x \to -\infty$

F-ja nema K.A.



Poslije ovog koraka počinjemo skicirati grafik.

rast i opadanje

$$y' = \left(\frac{2e^{2x}+1}{e^{2x}-2}\right)' = \frac{2e^{2x}\cdot 2(e^{2x}-2)-(2e^{2x}+1)\cdot e^{2x}\cdot 2}{(e^{2x}-2)^2} =$$

$$\frac{\cancel{4e^{4x}}-8e^{2x}\cancel{-4e^{4x}}-2e^{2x}}{(e^{2x}-2)^2} = (-10)\frac{e^{2x}}{(e^{2x}-2)^2}$$

$e^{2x}>0 \ \forall x\in\mathbb{R}$

$(e^{2x}-2)^2>0 \ \forall x\in\mathbb{R}$

Kako je $y'<0 \ \forall x\in\mathbb{R}$ to y opada ($\searrow$) za $\forall x\in D$

ekstremi f-je

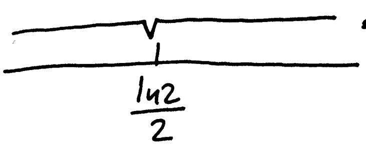
Kako f-ja stalno opada, f-ja nema ekstrem.

prevojne tačke i intervali konveksnosti i konkavnosti

$$y'' = \left((-10)\frac{e^{2x}}{(e^{2x}-2)^2}\right)' = (-10)\frac{e^{2x}\cdot 2(e^{2x}-2)^{\cancel{2}} - e^{2x}\cdot 2\cancel{(e^{2x}-2)}\cdot e^{2x}\cdot 2}{(e^{2x}-2)^{\cancel{4}\,3}} =$$

$$= (-10)\frac{2e^{4x}-4e^{2x}-4e^{4x}}{(e^{2x}-2)^3} = (-10)\frac{-2e^{4x}-4e^{2x}}{(e^{2x}-2)^3} = 20\cdot\frac{e^{4x}+2e^{2x}}{(e^{2x}-2)^3}$$

$e^{4x}+2e^{2x}>0 \ \forall x\in\mathbb{R}$

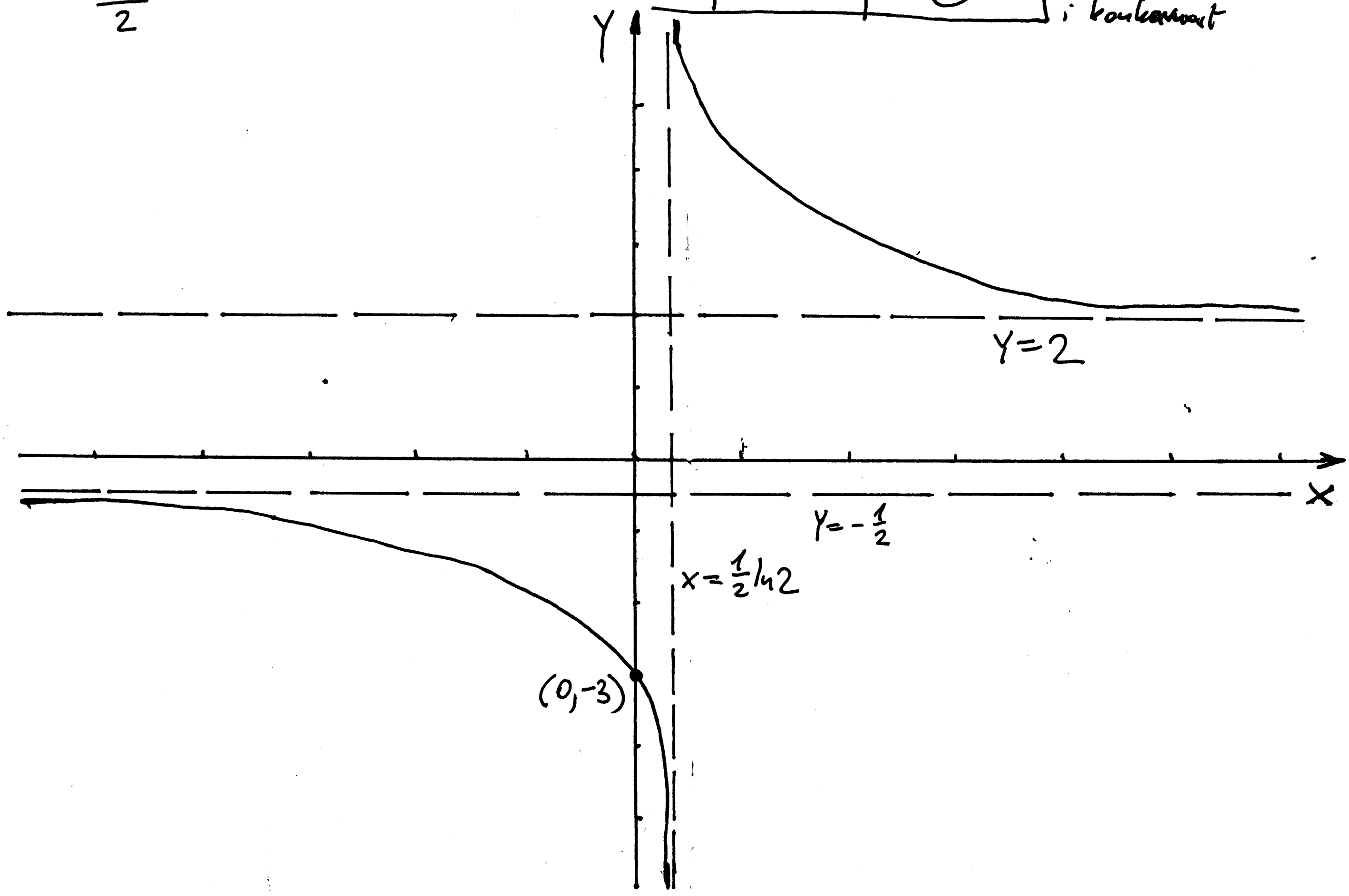


← prekidi y + nule y''

x	$(-\infty, \frac{\ln 2}{2})$	$(\frac{\ln 2}{2}, +\infty)$
y''	$-$	$+$
y	$\frown$	$\smile$

konveksnost i konkavnost

f-ja nema P_0T_0



(#) Izračunati integral $\int \frac{dx}{2+3\sin 2x+4\cos^2 x}$.

Rj.

$$\boxed{\begin{aligned}2+3\sin 2x+4\cos^2 x &= 2\cdot 1+3\cdot 2\sin x\cos x+4\cos^2 x=\\ &=2(\sin^2 x+\cos^2 x)+6\sin x\cos x+4\cos^2 x=\\ &=2\sin^2 x+6\sin x\cos x+6\cos^2 x=2(\sin^2 x+3\sin x\cos x+3\cos^2 x)\end{aligned}}$$

Uvodimo smjenu $\operatorname{tg} x=t$

$x=\operatorname{arctg} t$

$dx=\frac{dt}{1+t^2}$

$$\cos^2 x=\frac{\cos^2 x}{1}=\frac{\cos^2 x}{\sin^2 x+\cos^2 x}\begin{matrix}/:\cos^2 x\\ /:\cos^2 x\end{matrix}=\frac{1}{\operatorname{tg}^2 x+1}=\frac{1}{1+t^2}$$

$$\sin 2x=2\sin x\cos x=\frac{2\sin x\cos x}{\sin^2 x+\cos^2 x}\begin{matrix}/:\cos^2 x\\ /:\cos^2 x\end{matrix}=$$

$$=\frac{2\operatorname{tg} x}{\operatorname{tg}^2 x+1}=\frac{2t}{t^2+1}$$

Sad imamo

$$\int\frac{dx}{2+3\sin 2x+4\cos^2 x}=\left|\begin{matrix}\operatorname{tg} x=t\\ \cos^2 x=\frac{1}{t^2+1}\\ \sin 2x=\frac{2t}{t^2+1}\\ dx=\frac{dt}{1+t^2}\end{matrix}\right|=\int\frac{\frac{dt}{1+t^2}}{2+3\cdot\frac{2t}{t^2+1}+4\cdot\frac{1}{t^2+1}}=$$

$$=\int\frac{\frac{dt}{\cancel{t^2+1}}}{\frac{2(t^2+1)+6t+4}{\cancel{t^2+1}}}=\int\frac{dt}{2t^2+6t+6}=\frac{1}{2}\int\frac{dt}{t^2+3t+3}$$

$$=\left|\begin{matrix}t^2+2\cdot t\cdot\frac{3}{2}+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+3=\\ =\left(t+\frac{3}{2}\right)^2+\frac{3}{4}\\ \frac{12}{4}-\frac{9}{4}\end{matrix}\right|=\frac{1}{2}\int\frac{dt}{\left(t+\frac{3}{2}\right)^2+\frac{3}{4}}=\left|\begin{matrix}t+\frac{3}{2}=\frac{\sqrt{3}}{2}s\\ dt=\frac{\sqrt{3}}{2}ds\\ s=\frac{2t+3}{\sqrt{3}}\end{matrix}\right|=\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\int\frac{ds}{\frac{3}{4}s^2+\frac{3}{4}}$$

$$=\frac{\sqrt{3}}{4}\cdot\frac{4}{3}\int\frac{ds}{s^2+1}=\frac{\sqrt{3}}{3}\operatorname{arctg} s+C=\frac{\sqrt{3}}{3}\operatorname{arctg}\frac{2t+3}{\sqrt{3}}+C=\frac{\sqrt{3}}{3}\operatorname{arctg}\frac{2\operatorname{tg}x+3}{\sqrt{3}}+C$$

(#) Izračunati integral $\int \frac{\sin x + \cos x}{\sin x + 2\cos x} dx$

Rj. $\int \frac{\sin x + \cos x \quad /:\cos x}{\sin x + 2\cos x \quad /:\cos x} dx = \int \frac{\operatorname{tg} x + 1}{\operatorname{tg} x + 2} dx = \begin{vmatrix} \operatorname{tg} x = t \\ x = \operatorname{arctg} t \\ dx = \frac{dt}{1+t^2} \end{vmatrix} =$

$= \int \frac{t+1}{t+2} \cdot \frac{1}{1+t^2} dt = \int \frac{t+1}{(t+2)(1+t^2)} dt$

$\frac{t+1}{(t+2)(t^2+1)} = \frac{A}{t+2} + \frac{Bt+C}{t^2+1} \quad / (t+2)(t^2+1)$

$t+1 = A(t^2+1) + (Bt+C)(t+2)$

$t+1 = A(t^2+1) + B(t^2+2t) + C(t+2)$

$A + B = 0 \Rightarrow A = -B$

$2B + C = 1$

$A + 2C = 1 \Rightarrow A = 1-2C$

$A = -B$

$A = 1-2C$

$-B = 1-2C \quad /(-1)$

$B = 2C-1$

$2B + C = 1$

$2(2C-1) + C = 1$

$4C - 2 + C = 1$

$5C = 3$

$C = \frac{3}{5}, \quad A = 1 - \frac{6}{5} = -\frac{1}{5}$

$B = \frac{1}{5}$

$\int \frac{t+1}{(t+2)(t^2+1)} dt = \int \frac{-\frac{1}{5}}{t+2} dt + \int \frac{\frac{1}{5}t + \frac{3}{5}}{t^2+1} dt = -\frac{1}{5} \ln|t+2| + \frac{1}{5} \int \frac{t+3}{t^2+1} dt$

$= -\frac{1}{5} \ln|t+2| + \frac{1}{5} \int \frac{t\,dt}{t^2+1} + \frac{3}{5} \int \frac{dt}{t^2+1} = \begin{vmatrix} t^2+1 = s \\ 2t\,dt = ds \\ t\,dt = \frac{ds}{2} \end{vmatrix} =$

$= -\frac{1}{5} \ln|t+2| + \frac{1}{10} \ln|s| + \frac{3}{5} \operatorname{arctg} t + C =$

$= -\frac{1}{5} \ln|\operatorname{tg} x + 2| + \frac{1}{10} \ln|\operatorname{tg}^2 x + 1| + \frac{3}{5} \operatorname{arctg}(\operatorname{tg} x) + C$