Univerzitet u Zenici
Fakultet za metalurgiju i materijale

Zadaci sa pismenog ispita rađenog 29.04.2009. iz predmeta MATEMATIKA 1

1. Diskutovati rješenja sistema u zavisnosti od parametra λ

$$\begin{aligned} x+y+z &= 0 \\ x+3y+(\lambda+3)z &= 1 \\ (\lambda-4)x+y+4z &= 1 \end{aligned} .$$

2. Data su tjemena paralelograma $A(3,2,5)$, $B(4,7,5)$ i $C(-\lambda,7,5+\lambda)$. Odredite četvrto tjeme D i odredite za koju vrijednost parametra λ je $|\overrightarrow{AC}| = 3\sqrt{6}$.

3. Odredite parametar λ u jednačini prave $\dfrac{x+2}{\lambda} = \dfrac{y-3}{1} = \dfrac{z+1}{-1}$ da bi se sjekla sa pravom $\dfrac{x-2}{-11} = \dfrac{y-1}{-5} = \dfrac{z-1}{2}$ i u tom slučaju naći presječnu tačku i ugao između pravih.

4. Izračunati: $\lim\limits_{n\to\infty} \dfrac{(n-1)(2n-2)(2n-3)}{3n^3}$.

1. Diskutovati rješenja sistema u zavisnosti od parametra λ

$$\begin{aligned} x + y + z &= 0 \\ x + 3y + (\lambda+3)z &= 1 \\ (\lambda-4)x + y + 4z &= 1 \end{aligned}$$

Rj.

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & \lambda+3 \\ \lambda-4 & 1 & 4 \end{vmatrix} \underline{\underline{\substack{I_k - II_k \\ III_k - II_k}}} \begin{vmatrix} 0 & 1 & 0 \\ -2 & 3 & \lambda \\ \lambda-5 & 1 & 3 \end{vmatrix} = - \begin{vmatrix} -2 & \lambda \\ \lambda-5 & 3 \end{vmatrix} = -(-6-\lambda^2+5\lambda) =$$
$$= \lambda^2 - 5\lambda + 6$$
$$= (\lambda-2)(\lambda-3)$$

$$D_x = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 3 & \lambda+3 \\ 1 & 1 & 4 \end{vmatrix} \underline{\underline{III_k - II_k}} \begin{vmatrix} 0 & 1 & 0 \\ 1 & 3 & \lambda \\ 1 & 1 & 3 \end{vmatrix} = - \begin{vmatrix} 1 & \lambda \\ 1 & 3 \end{vmatrix} = -(3-\lambda) = \lambda - 3$$

$$D_y = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & \lambda+3 \\ \lambda-4 & 1 & 4 \end{vmatrix} \underline{\underline{I_k - III_k}} \begin{vmatrix} 0 & 0 & 1 \\ -\lambda-2 & 1 & \lambda+3 \\ \lambda-8 & 1 & 4 \end{vmatrix} = \begin{vmatrix} -\lambda-2 & 1 \\ \lambda-8 & 1 \end{vmatrix} = -\lambda-2-\lambda+8 =$$
$$= -2\lambda+6 = -2(\lambda-3)$$

$$D_z = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ \lambda-4 & 1 & 1 \end{vmatrix} \underline{\underline{III_v - II_v}} \begin{vmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ \lambda-5 & -2 & 0 \end{vmatrix} = - \begin{vmatrix} 1 & 1 \\ \lambda-5 & -2 \end{vmatrix} = -(-2-\lambda+5) =$$
$$= \lambda - 3$$

Diskusija

1° $\lambda \neq 2$ i $\lambda \neq 3$

$D \neq 0$, sistem ima jedinstveno rješenje

$$x = \frac{D_x}{D} = \frac{\lambda-3}{(\lambda-2)(\lambda-3)} = \frac{1}{\lambda-2}$$

$$y = \frac{D_y}{D} = \frac{-2(\lambda-3)}{(\lambda-2)(\lambda-3)} = \frac{-2}{\lambda-2} \qquad \left(\frac{1}{\lambda-2}, \frac{-2}{\lambda-2}, \frac{1}{\lambda-2}\right)$$

$$z = \frac{D_z}{D} = \frac{\lambda-3}{(\lambda-2)(\lambda-3)} = \frac{1}{\lambda-2}$$

2° $\lambda = 2$

$D = 0$, $D_x = -1 \neq 0 \Rightarrow$ sistem nema nijedno rješenje

3° $\lambda = 3$

$D = D_x = D_y = D_z = 0$

sistem postaje

$$\begin{aligned} x + y + z &= 0 \quad (1) \\ x + 3y + 6z &= 1 \quad (2) \\ -x + y + 4z &= 1 \quad (3) \end{aligned}$$

$(2)-(1): \quad 2y+5z=1$

$(1)+(3): \quad \underline{2y+5z=1}$

$2y=1-5z$

$y=\frac{1-5z}{2}$

$x+y+z=0$

$x=-y-z$

$x=-\frac{1-5z}{2}-z$

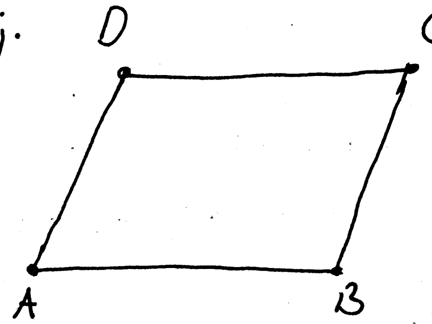
$x=\frac{-1+5z-2z}{2}$

$x=\frac{3z-1}{2}$

sistem ima ∞ mnogo rješenja oblika $\left(\frac{3t-1}{2}, \frac{1-5t}{2}, t\right)$, $t \in R$

2. Data su tjemena paralelograma $A(3,2,5)$, $B(4,7,5)$ i $C(-\lambda, 7, 5+\lambda)$. Odredite četvrto teme D i odredite za koju vrijednost λ je $|\overrightarrow{AC}|=3\sqrt{6}$.

Rj.



$\overrightarrow{AB}=\overrightarrow{DC}$

$D(x,y,z)$
$C(-\lambda, 7, 5+\lambda)$
$\Downarrow$
$\overrightarrow{DC}=(-\lambda-x, 7-y, 5+\lambda-z)$

$A(3,2,5)$
$B(4,7,5)$ $\Rightarrow \overrightarrow{AB}=(1,5,0)$

$-\lambda-x=1$
$7-y=5$
$5+\lambda-z=0$

$x=-\lambda-1$
$y=2$
$z=5+\lambda$

$D(-\lambda-1, 2, 5+\lambda)$

$A(3,2,5)$
$C(-\lambda, 7, 5+\lambda)$ $\Rightarrow \overrightarrow{AC}=(-\lambda-3, 5, \lambda)$

$|\overrightarrow{AC}|=\sqrt{\lambda^2+6\lambda+9+25+\lambda^2}$

$|\overrightarrow{AC}|=3\sqrt{6}$

$\sqrt{2\lambda^2+6\lambda+34}=3\sqrt{6} \quad /^2$

$2\lambda^2+6\lambda+34=54$

$2\lambda^2+6\lambda-20=0 \quad /:2$

$\lambda^2+3\lambda-10=0$

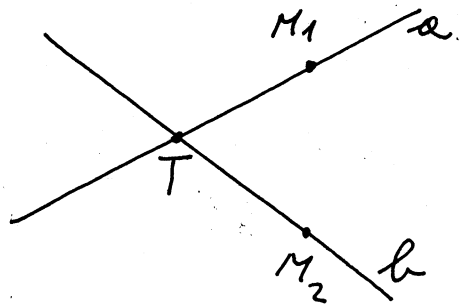
$(\lambda+5)(\lambda-2)=0$

Za $\lambda=-5$ ili $\lambda=2$ je $|\overrightarrow{AC}|=3\sqrt{6}$.

3. Odredite parametar λ u jednačini prave $\frac{x+2}{\lambda}=\frac{y-3}{1}=\frac{z+1}{-1}$ da bi se sjekla sa pravom $\frac{x-2}{-11}=\frac{y-1}{-5}=\frac{z-1}{2}$ i u tom slučaju naći presječnu tačku pravih i ugao između pravih.

Rj. $a: \frac{x+2}{\lambda} = \frac{y-3}{1} = \frac{z+1}{-1}$, $\vec{p}_a(\lambda, 1, -1)$, $M_1(-2,3,-1)$, $M_1 \in a$

$b: \frac{x-2}{-11} = \frac{y-1}{-5} = \frac{z-1}{2}$, $\vec{p}_b(-11,-5,2)$, $M_2(2,1,1)$, $M_2 \in b$



potreban uslov da se prave sijeku je $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

$$\begin{vmatrix} 4 & -2 & 2 \\ \lambda & 1 & -1 \\ -11 & -5 & 2 \end{vmatrix} = 0 \xRightarrow{I_v + II_v \cdot 2} \begin{vmatrix} 2\lambda+4 & 0 & 0 \\ \lambda & 1 & -1 \\ -11 & -5 & 2 \end{vmatrix} = 0$$

$$\Rightarrow (2\lambda+4)\begin{vmatrix} 1 & -1 \\ -5 & 2 \end{vmatrix} = 0 \Rightarrow (2\lambda+4)\cdot(2-5) = 0 \Rightarrow -6(\lambda+2) = 0$$

$$\lambda = -2$$

Za vrijednost $\lambda=-2$ prave a i b se sijeku.

$\frac{x+2}{-2} = \frac{y-3}{1} = \frac{z+1}{-1}$ $(=t)$

$x+2=-2t$
$y-3=t$
$z+1=-t$

$x=-2t-2$
$y=t+3$
$z=-t-1$

$\frac{x-2}{-11} = \frac{y-1}{-5} = \frac{z-1}{2}$ $(=s)$

$x-2=-11s$
$y-1=-5s$
$z-1=2s$

$x=-11s+2$
$y=-5s+1$
$z=2s+1$

$-2t-2=-11s+2$
$t+3=-5s+1$
$-t-1=2s+1$

$-2t+11s=4$ (1)
$t+5s=-2$ (2)
$-t-2s=2$ (3)

(2)+(3): $3s=0$
$s=0$

uvrstimo u (1)

$-2t=4 \Rightarrow t=-2$

za $s=0$ ili $t=-2$ $x=2$, $y=1$, $z=1$

Presječna tačka pravih a i b je $T(2,1,1)$.

$\vec{p}_a \cdot \vec{p}_b = |\vec{p}_a| \cdot |\vec{p}_b| \cdot \cos\measuredangle(\vec{p}_a, \vec{p}_b)$

$\vec{p}_a(-2,1,-1)$

$\cos\measuredangle(\vec{p}_a, \vec{p}_b) = \frac{\vec{p}_a \cdot \vec{p}_b}{|\vec{p}_a| \cdot |\vec{p}_b|}$

$\vec{p}_b(-11,-5,2)$

$$\vec{p}_a \cdot \vec{p}_b = 22 - 5 - 2 = 15$$

$$|\vec{p}_a| = \sqrt{4+1+1} = \sqrt{6}$$

$$|\vec{p}_b| = \sqrt{121+25+4} = \sqrt{150} = \sqrt{5\cdot 30} = \sqrt{5\cdot 5\cdot 6} = 5\sqrt{6}$$

$$\cos\angle(\vec{p}_a, \vec{p}_b) = \frac{15}{5\sqrt{6}\cdot\sqrt{6}} = \frac{15}{30} = \frac{1}{2} \Rightarrow \angle(\vec{p}_a, \vec{p}_b) = \frac{\pi}{3}$$

Ugao između pravih je 60°.

4. Izračunati: $\lim\limits_{n\to\infty} \dfrac{(n-1)(2n-2)(2n-3)}{3n^3}$.

Rj.

$$\lim_{n\to\infty} \frac{(n-1)(2n-2)(2n-3)}{3n^3} = \lim_{n\to\infty} \frac{4n^3 - 12n^2 + 11n - 3}{3n^3} \begin{matrix} /:n^3 \\ /:n^3 \end{matrix} = \frac{4}{3}$$