Univerzitet u Zenici
Pedagoški fakultet
Odsjek: Matematika i informatika
Zenica, 03.01.2010.

Pismeni ispit iz predmeta **Analiza 3**

1. Data je funkcija

$$\Lambda(t) = \begin{cases} 1 - |t|, & \text{ako je } |t| < 0 \\ 0, & \text{ako je } 1 \leq |t| \leq \pi \end{cases} .$$

Grafički predstaviti funkciju, razviti je u Fourierov red u intervalu $[-\pi, \pi]$ i naći sumu reda

$$\sum_{n=1}^{\infty} \frac{1 - (-1)^n}{n^2} .$$

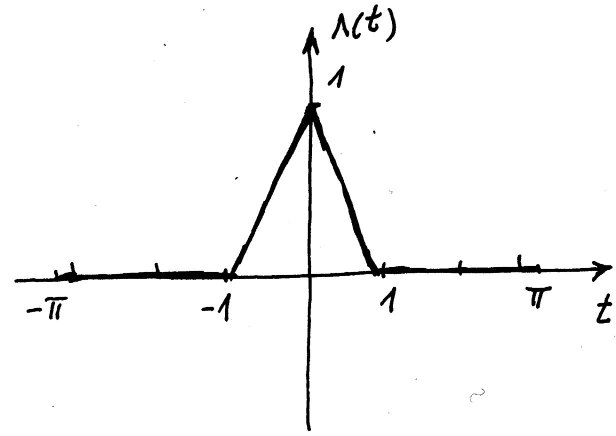
2. Funkciju $f(x, y) = arc\,tg \dfrac{x - y}{1 + xy}$ razviti u Tejlorov red do članova četvrtog reda u okolini tačke $(0, 0)$. Prikazati izgled opšteg člana.

3. Izračunati dvostruki integral $\iint_D (x^2 + y^2)\,dx\,dy$ gdje je $D = \{(x, y) \in \mathbb{R} \mid x^2 + y^2 \leq \frac{2}{3}(x + 2y)\}$.

4. Izračunati površinski integral $\iint_S 3z\,dS$ gdje je S površina paraboloida $z = 2 - (x^2 + y^2)$ iznad xy-ravni.

(Za uočene greške pisati na **infoarrt@gmail.com**)

(#) Data je f-ja $\Lambda(t)=\begin{cases}1-|t|, & |t|<1\\ 0, & 1\le |t|\le \pi\end{cases}$. Grafički predstaviti f-ju, razviti je u Fourierov red u intervalu $[-\pi,\pi]$ i naći sumu reda $\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2}$.



Rj. Za $t\ge 0$ $\Lambda(t)=\begin{cases}1-t, & 0\le t<1\\ 0, & 1\le t\le\pi\end{cases}$

Za $t<0$ $\Lambda(t)=\begin{cases}1+t, & -1<t<0\\ 0, & -\pi\le t\le -1\end{cases}$

Kako f-ja $\Lambda(t)$ ima vrijednost nula (0) u intervalima $[-\pi,-1)$ i $[1,\pi]$ to je dovoljno f-ju razviti u intervalu $(-1,1)$

Za $[-\ell,\ell]$ imamo sledeće formule

Furijerov red: $f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos\frac{n\pi x}{\ell}+b_n\sin\frac{n\pi x}{\ell}\right)$

Furijerovi koeficijenti: $a_0=\frac{1}{\ell}\int_{-\ell}^{\ell}f(x)\,dx$, $a_n=\frac{1}{\ell}\int_{-\ell}^{\ell}f(x)\cos\frac{n\pi x}{\ell}dx$ $n=1,2,\dots$

i $b_n=\frac{1}{\ell}\int_{-\ell}^{\ell}f(x)\sin\frac{n\pi x}{\ell}dx$ $n=1,2,\dots$ Primjetimo da je f-ja $\Lambda(t)$ parna (simetrična u odnosu na y-osu) pa odmah možemo zaključiti da su $b_n=0$, $n=1,2,\dots$

$$a_0=\frac{2}{1}\int_0^1\Lambda(x)\,dx=2\int_0^1(1-x)\,dx=2\left(x\Big|_0^1-\frac{1}{2}x^2\Big|_0^1\right)=2\cdot\frac{1}{2}=1$$

$$a_n=\frac{2}{\ell}\int_0^{\ell}\Lambda(x)\cos\frac{n\pi x}{\ell}dx=2\int_0^1(1-x)\cos n\pi x\,dx=\begin{vmatrix}u=1-x & dv=\cos n\pi x\\ du=-dx & v=\frac{1}{n\pi}\sin n\pi x\end{vmatrix}$$

$$=2\cdot\frac{1}{n\pi}\left[(1-x)\sin n\pi x\right]\Big|_0^1+\frac{2}{n\pi}\int_0^1\sin n\pi x\,dx=0+\frac{2}{n\pi}\cdot\frac{-1}{n\pi}\cos n\pi x\Big|_0^1=\frac{-2}{n^2\pi^2}\left((-1)^n-1\right)$$

$$\Lambda(x)=\frac{1}{2}+\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2}\cos n\pi x,\quad x\in(-1,1)$$

Razvoj f-je u Fourierov red u intervalu $[-1,1]$

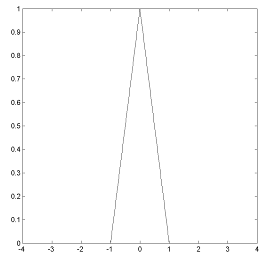
$(\Lambda(x)=0$, za $x\in[-\pi,-1]\cup[1,\pi])$

Za $x=0$ imamo: $$1 = \frac{1}{2} + \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2}$$

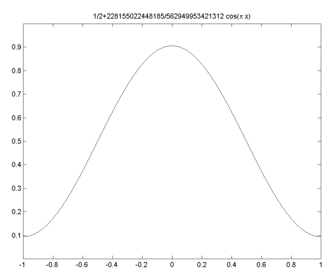
$$\frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2} = \frac{1}{2} \quad \Rightarrow \quad \sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2} = \frac{\pi^2}{4}$$ vrijednost tražene sume

DODATAK:

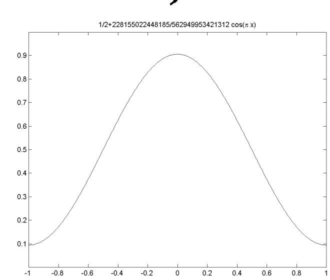
Pustimo MatLab da nacrta f-ju $\Lambda(t) = \begin{cases} 1-|t|, & |t|<1 \\ 0, & 1\le |t| \le \pi \end{cases}$



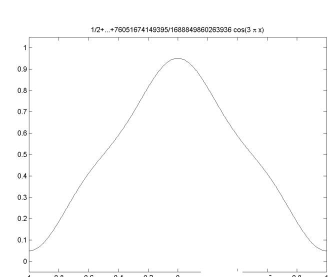
Grafički predstavimo sljedeće trigonometrijske polinome, koje smo dobili iz pronađenog Fourierovog reda, na intervalu $[-1,1]$:



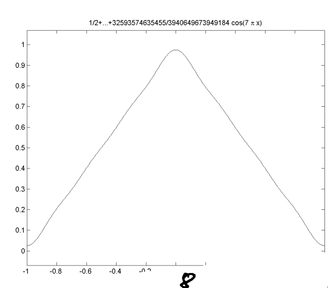
$$P_1(x) = \frac{1}{2} + \frac{2}{\pi^2}\cdot\frac{1-(-1)^1}{1^2}\cdot\cos\pi x$$



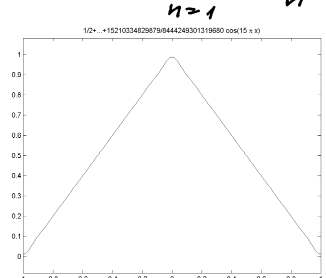
$$P_2(x) = \frac{1}{2} + \frac{2}{\pi^2}\sum_{n=1}^{2}\frac{1-(-1)^n}{n^2}\cos n\pi x$$



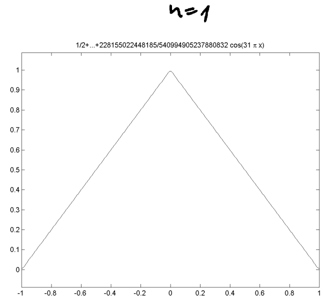
$$P_4(x) = \frac{1}{2} + \frac{2}{\pi^2}\sum_{n=1}^{4}\frac{1-(-1)^n}{n^2}\cos n\pi x$$



$$P_8(x) = \frac{1}{2} + \frac{2}{\pi^2}\sum_{n=1}^{8}\frac{1-(-1)^n}{n^2}\cos n\pi x$$



$$P_{16}(x) = \frac{1}{2} + \frac{2}{\pi^2}\sum_{n=1}^{16}\frac{1-(-1)^n}{n^2}\cos n\pi x$$



$$P_{32}(x) = \frac{1}{2} + \frac{2}{\pi^2}\sum_{n=1}^{32}\frac{1-(-1)^n}{n^2}\cos n\pi x$$

Šta možemo primijetiti? Kako bi graf f-je $P_{32}(x)$ izgledao na intervalu $[-\pi,\pi]$?

(#) F-ju $f(x,y)=\operatorname{arctg}\dfrac{x-y}{1+xy}$ razviti u Tejlorov red do članova 4. reda u okolini tačke $(0,0)$. Prikazati izgled opšteg člana.

Rj. F-ja $z=f(x,y)$ razložena po formuli Tejlora u okolini tačke (ρ_1,ρ_2):

$$f(x,y)=f(\rho_1,\rho_2)+\sum_{k=1}^{\infty}\frac{1}{k!}\left((x-\rho_1)\frac{\partial}{\partial x}+(y-\rho_2)\frac{\partial}{\partial y}\right)^k f(\rho_1,\rho_2)=$$

$$=f(\rho_1,\rho_2)+\frac{1}{1!}\left[\frac{\partial f(\rho_1,\rho_2)}{\partial x}(x-\rho_1)+\frac{\partial f(\rho_1,\rho_2)}{\partial y}(y-\rho_1)\right]+\frac{1}{2!}\left[\frac{\partial^2 f(\rho_1,\rho_2)}{\partial x^2}(x-\rho_1)^2+\right.$$

$$\left.+\frac{\partial^2 f(\rho_1,\rho_2)}{\partial x\partial y}(x-\rho_1)(y-\rho_2)+\frac{\partial^2 f(\rho_1,\rho_2)}{\partial y^2}(y-\rho_2)^2\right]+\frac{1}{3!}\left[\frac{\partial^3 f(\rho_1,\rho_2)}{\partial x^3}(x-\rho_1)^3+3\frac{\partial^3 f(\rho_1,\rho_2)}{\partial x^2\partial y}(x-\rho_1)^2(y-\rho_2)\right.$$

$$\left.+3\frac{\partial^3 f(\rho_1,\rho_2)}{\partial x\partial y^2}(x-\rho_1)(y-\rho_2)^2+\frac{\partial^3 f(\rho_1,\rho_2)}{\partial y^2}(y-\rho_2)^3\right]+\frac{1}{4!}\cdot\left[\frac{\partial^4 f(\rho_1,\rho_2)}{\partial x^4}(x-\rho_1)^4+\ldots\right.$$

$$\frac{\partial f}{\partial x}=\frac{1}{1+\left(\frac{x-y}{1+xy}\right)^2}\cdot\frac{(1+xy)-(x-y)\cdot y}{(1+xy)^2}=\frac{(1+xy)^2}{(1+xy)^2+(x-y)^2}\cdot\frac{1+xy-xy+y^2}{(1+xy)^2}=\frac{1+y^2}{1+\underline{2xy}+x^2y^2+x^2\underline{-2xy}+y^2}=$$

$$=\frac{1+y^2}{1+x^2+y^2+x^2y^2}=\frac{1+y^2}{1+x^2+y^2(1+x^2)}=\frac{1+y^2}{(1+x^2)(1+y^2)}=\underline{\underline{\frac{1}{1+x^2}}}$$

$$\frac{\partial f}{\partial x}(0,0)=1,\quad \frac{\partial f}{\partial y}=\frac{1}{1+\left(\frac{x-y}{1+xy}\right)^2}\cdot\frac{(-1)(1+xy)-(x-y)x}{(1+xy)^2}=\frac{(1+xy)^2}{(1+xy)^2+(x-y)^2}\cdot\frac{-1-xy-x^2+xy}{(1+xy)^2}$$

$$=\frac{(-1)(1+x^2)}{1+\underline{2xy}+x^2y^2+x^2\underline{-2xy}+y^2}=\frac{(-1)(1+x^2)}{(1+x^2)(1+y^2)}=\underline{\underline{\frac{-1}{1+y^2}}},\quad \frac{\partial f}{\partial y}(0,0)=-1$$

$$\underline{\underline{\frac{\partial^2 f}{\partial x^2}=\frac{-2x}{(1+x^2)^2}}},\quad \frac{\partial^2 f}{\partial x^2}(0,0)=0,\quad \frac{\partial^2 f}{\partial x\partial y}=0,\quad \underline{\underline{\frac{\partial^2 f}{\partial y^2}=\frac{2y}{(1+y^2)^2}}},\quad \frac{\partial^2 f}{\partial y^2}(0,0)=0$$

$$\frac{\partial^3 f}{\partial x^3}=-2\,\frac{(1+x^2)^2-x\cdot 2(1+x^2)\cdot 2x}{(1+x^2)^4}=-2\,\frac{1+x^2-4x^2}{(1+x^2)^3}=\underline{\underline{-2\,\frac{1-3x^2}{(1+x^2)^3}}},\quad \frac{\partial^3 f}{\partial x^3}(0,0)=-2$$

$$\frac{\partial^3 f}{\partial x^2\partial y}=0,\quad \frac{\partial^3 f}{\partial x\partial y^2}=0,\quad \frac{\partial^3 f}{\partial y^3}=2\,\frac{(1+y^2)^2-y\cdot 2(1+y^2)\cdot 2y}{(1+y^2)^4}=2\,\frac{1+y^2-4y^2}{(1+y^2)^3}=\underline{\underline{2\,\frac{1-3y^2}{(1+y^2)^3}}}$$

$$\frac{\partial^3 f}{\partial y^3}(0,0)=2,\quad \frac{\partial^4 f}{\partial x^4}=(-2)\cdot\frac{-6x(1+x^2)^3-(1-3x^2)\cdot 3(1+x^2)^2\cdot 2x}{(1+x^2)^6}=(-2)\frac{-6x-6x^3-6x+18x^3}{(1+x^2)^4}$$

$$=(-2)\,\frac{-12x+12x^3}{(1+x^2)^4}=(-2)(-12)\,\frac{x-x^3}{(1+x^2)^4}=\underline{\underline{-24\,\frac{x(x^2-1)}{(x^2+1)^4}}},\quad \frac{\partial^4 f}{\partial x^4}(0,0)=0,\quad \frac{\partial^4 f}{\partial x^3\partial y}=0$$

$$\frac{\partial^4 f}{\partial x^2 \partial y^2} = 0, \quad \frac{\partial^4 f}{\partial x \partial y^3} = 0, \quad \frac{\partial^4 f}{\partial y^4} = 2\,\frac{-6y(1+y^2)^3 - (1-3y^2)3\cdot(1+y^2)^2\cdot 2y}{(1+y^2)} =$$

$$= 2\,\frac{-6y-6y^3-6y+18y^3}{(1+y^2)^4} = 2\,\frac{-12y+12y^3}{(1+y^2)^4} = 2\cdot 12\,\frac{-y+y^3}{(1+y^2)^4} = 24\,\frac{y(y^2-1)}{(y^2+1)^4}$$

$$\frac{\partial^4 f}{\partial y^4}(0,0) = 0, \quad f(0,0) = \operatorname{arctg} 0 = 0$$

$$f(x,y) = \frac{1}{1!}(x-y) + \frac{1}{2!}(0+0+0) + \frac{1}{3!}\left((-2)x^3 + 2y^3\right) + \frac{1}{4!}\cdot 0 + \ldots$$

$$= x - y + \frac{-1}{3}(x^3-y^3) + \ldots = x - y + \frac{(-1)^1}{3}(x^3-y^3) + \frac{(-1)^2}{5}(x^5-y^5)$$

$$+ \ldots + \frac{(-1)^n}{2n+1}\left(x^{2n+1} - y^{2n+1}\right) + \ldots$$ f-ja $f(x,y)$ razložena po formuli Tejlora

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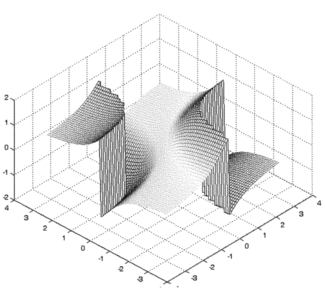
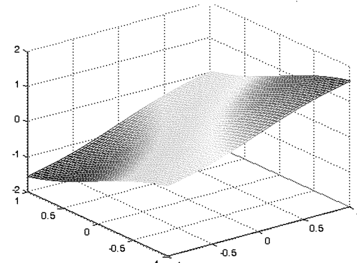
Dodatak.

Grafički prikažimo f-ju $f(x,y) = \operatorname{arctg} \frac{x-y}{1+xy}$ na intervalu



na intervalu [-pi,pi]×[-pi,pi]



na intervalu (-1,1)×(-1,1)

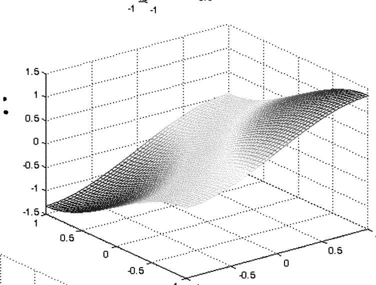
Grafički prikažimo sljedeće polinome:

$$f(x,y) = \sum_{n=0}^{1} \frac{(-1)^n}{2n+1}\left(x^{2n+1} - y^{2n+1}\right)$$

$$f(x,y) = \sum_{n=0}^{2} \frac{(-1)^n}{2n+1}\left(x^{2n+1} - y^{2n+1}\right)$$

$$f(x,y) = \sum_{n=0}^{4} \frac{(-1)^n}{2n+1}\left(x^{2n+1} - y^{2n+1}\right)$$

$$f(x,y) = \sum_{n=0}^{10} \frac{(-1)^n}{2n+1}\left(x^{2n+1} - y^{2n+1}\right)$$



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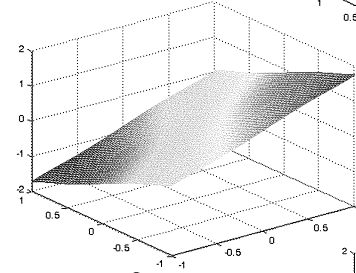

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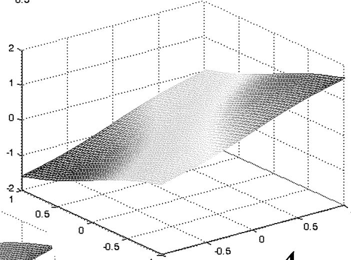

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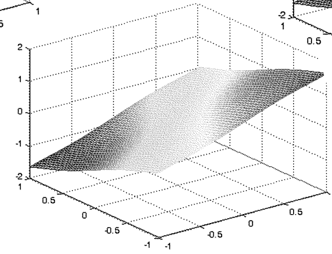
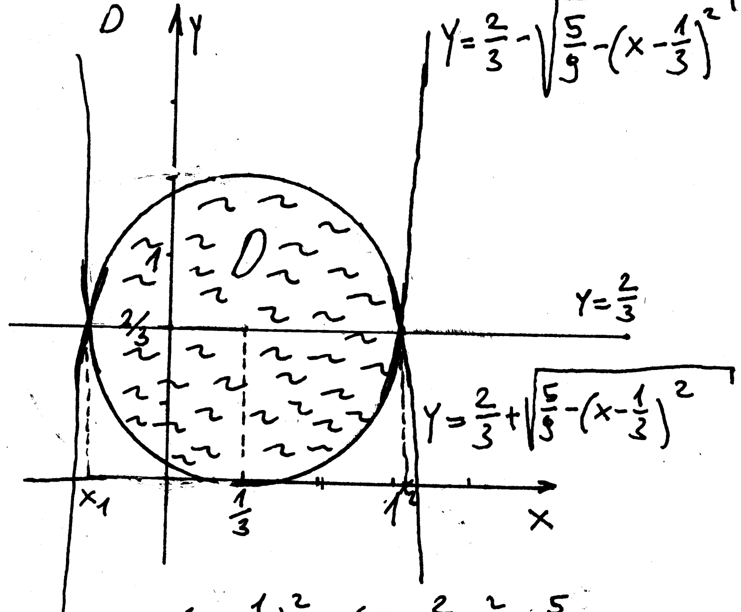


3

Šta možemo primjetiti. Šta bi se desilo da smo uzeli interval $[-\pi,\pi]\times[-\pi,\pi]$ (Kako bi izgledao graf?).

#) Izračunati dvostruki integral $I=\iint\limits_D (x^2+y^2)\,dx\,dy$ gdje je

$$D=\left\{(x,y)\in\mathbb{R}^2 \;\middle|\; x^2+y^2 \le \frac{2}{3}(x+2y)\right\}.$$



Rj. Odredimo šta je oblast D.

$$x^2+y^2 \le \frac{2}{3}(x+2y)$$

$$x^2+y^2 \le \frac{2}{3}x+\frac{4}{3}y$$

$$x^2-\frac{2}{3}x+y^2-\frac{4}{3}y \le 0$$

$$x^2-2\cdot x\cdot\frac{1}{3}+\frac{1}{9}-\frac{1}{9}+y^2-2\cdot y\cdot\frac{2}{3}+\frac{4}{9}-\frac{4}{9} \le 0$$

$$\left(x-\frac{1}{3}\right)^2+\left(y-\frac{2}{3}\right)^2 \le \frac{5}{9}$$

D predstavlja unutrašnjost kruga s centrom u tački $\left(\frac{1}{3},\frac{2}{3}\right)$ poluprečnika $r=\frac{\sqrt{5}}{3}\approx 0,74$

$$\left(x-\frac{1}{3}\right)^2+\left(y-\frac{2}{3}\right)^2=\frac{5}{9}$$

$$y=\frac{2}{3}$$

$$\left(x-\frac{1}{3}\right)^2=\frac{5}{9}$$

$$x-\frac{1}{3}=\pm\frac{\sqrt{5}}{3}$$

$$x_{1,2}=\frac{1\pm\sqrt{5}}{3}$$

$$\left(y-\frac{2}{3}\right)^2=\frac{5}{9}-\left(x-\frac{1}{3}\right)^2$$

$$y=\frac{2}{3}\pm\sqrt{\frac{5}{9}-\left(x-\frac{1}{3}\right)^2}$$

I način: klasičan način

Nađimo presječnu tačku kruga i prave $y=\frac{2}{3}$

$$I=\iint\limits_D (x^2+y^2)\,dx\,dy=\int_{\frac{1-\sqrt{5}}{3}}^{\frac{1+\sqrt{5}}{3}}\left[\int_{\frac{2}{3}-\sqrt{\frac{5}{9}-\left(x-\frac{1}{3}\right)^2}}^{\frac{2}{3}+\sqrt{\frac{5}{9}-\left(x-\frac{1}{3}\right)^2}}(x^2+y^2)\,dy\right]dx=\ldots$$

NA KLASIČAN NAČIN OVO JE TEŠKO URADITI

II način: Uvedimo neku smjenu promjenljivih. Kako je dat krug uvedimo polarne koordinate.

$$x=a+r\cos\varphi \qquad y=b+r\sin\varphi$$

tj. $x=\frac{1}{3}+r\cos\varphi$, $y=\frac{2}{3}+r\sin\varphi$

$0\le\varphi\le 2\pi$, $0\le r\le\frac{\sqrt{5}}{3}$ ove vrijednosti čitamo sa slike

$dx\,dy=r\,dr\,d\varphi$.

Jakobijan

$$dx\,dy=|J|\,dr\,d\varphi$$

$$J=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial x}{\partial\varphi}\\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial\varphi}\end{vmatrix}=\begin{vmatrix}\cos\varphi & -r\sin\varphi\\ \sin\varphi & r\cos\varphi\end{vmatrix}=r$$

$$x^2+y^2=\left(\frac{1}{3}+r\cos\varphi\right)^2+\left(\frac{2}{3}+r\sin\varphi\right)^2=\frac{1}{9}+\frac{2}{3}r\cos\varphi+r^2\cos^2\varphi+\frac{4}{9}+\frac{4}{3}r\sin\varphi+r^2\sin^2\varphi=$$

$$=\frac{5}{9}+r^2+\frac{2}{3}r\cos\varphi+\frac{4}{3}r\sin\varphi$$

$$I=\iint\limits_D (x^2+y^2)\,dx\,dy=\iint\limits_{D'}\left(\frac{5}{9}+r^2+\frac{2}{3}r(\cos\varphi+2\sin\varphi)\right)r\,dr\,d\varphi=\iint\limits_{D'}\left(\frac{5}{9}r+r^3\right)dr\,d\varphi+$$

$$+\frac{2}{3}\iint\limits_{D'}r^2(\cos\varphi+2\sin\varphi)\,dr\,d\varphi,\quad \iint\limits_{D'}\left(\frac{5}{9}r+r^3\right)dr\,d\varphi=\int_0^{2\pi}\left[\int_0^{\frac{\sqrt{5}}{3}}\left(\frac{5}{9}r+r^3\right)dr\right]d\varphi=2\pi\cdot\left(\frac{5}{9}\cdot\frac{1}{2}r^2\Big|_0^{\frac{\sqrt{5}}{3}}+\right.$$

$$+\frac{1}{4}r^4\Big|_0^{\frac{\sqrt{5}}{3}}\Big) = 2\pi\left(\frac{5}{9\cdot 2}\cdot\frac{5}{9}+\frac{1}{4}\cdot\frac{5\cdot 5}{9\cdot 9}\right)=\pi\left(\frac{5^2}{9^2}+\frac{1}{2}\cdot\frac{5^2}{9^2}\right)=\frac{3}{2}\,\frac{5^2}{9^2}\pi=\frac{25}{54}\pi$$

$$\iint_{D'} r^2(\cos\varphi+2\sin\varphi)\,dr\,d\varphi = \int_0^{\sqrt{5}/3} r^2\left[\int_0^{2\pi}(\cos\varphi+2\sin\varphi)\,d\varphi\right] = \frac{r^3}{3}\Big|_0^{\sqrt{5}/3}\left(\sin\varphi\Big|_0^{2\pi}-2\cos\varphi\Big|_0^{2\pi}\right)=0$$

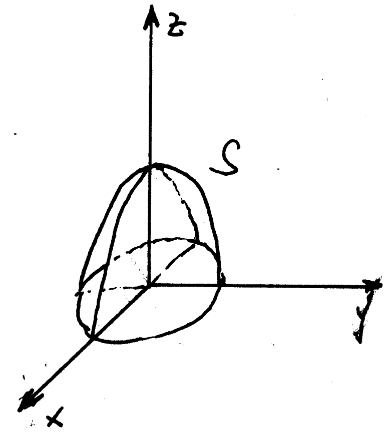
Prema tome $\displaystyle\iint_D (x^2+y^2)\,dx\,dy = \frac{25}{54}\pi$

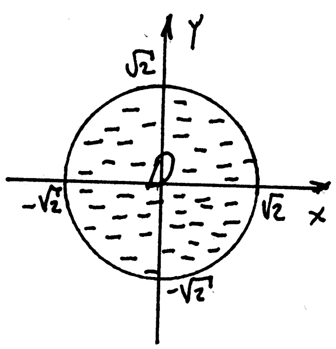
\# Izračunati površinski integral $\displaystyle\iint_S 3z\,dS$ gdje je S površina paraboloida $z=2-(x^2+y^2)$ iznad xy-ravni.

Rj: Neka je D projekcija površi S na xOy ravan. Tada

$$\iint_S f(x,y,z)\,dS = \iint_D f(x,y,z(x,y))\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dx\,dy$$

Pronađimo projekciju paraboloida $z=2-(x^2+y^2)$ na xOy ravan.





$z=0 \Rightarrow x^2+y^2=2$ krug sa centrom u tački $(0,0)$ poluprečnika $\sqrt{2}$

$$\frac{\partial z}{\partial x} = -2x$$

$$\frac{\partial z}{\partial y} = -2y$$

$$I=\iint_S 3z\,dS = 3\iint_D \left[2-(x^2+y^2)\right]\sqrt{1+4x^2+4y^2}\,dx\,dy$$

Da bi smo riješili ovaj dvostruki integral potrebno je uvesti smjenu promjenjivih.

Uvedimo polarne koordinate $x = r\cos\varphi$, $y = r\sin\varphi$

$$D': \begin{cases} 0\le\varphi\le 2\pi \\ 0\le r\le\sqrt{2} \\ dx\,dy = r\,dr\,d\varphi \end{cases}$$

ove granice čitamo sa slike

$$x^2+y^2 = r^2\cos^2\varphi + r^2\sin^2\varphi = r^2$$

$$1+4x^2+4y^2 = 1+4(x^2+y^2) = 1+4r^2$$

$$I = 3\iint_{D'}(2-r^2)\sqrt{1+4r^2}\cdot r\cdot dr\,d\varphi = 3\iint_{D'}2r\sqrt{1+4r^2}\,dr\,d\varphi - 3\iint_{D'}r^3\sqrt{1+4r^2}\,dr\,d\varphi$$

$$6\iint\limits_{D} r\sqrt{1+4r^2}\,dr\,d\varphi = 6\int_0^{2\pi}\left[\int_0^{\sqrt{2}} r\sqrt{1+4r^2}\,dr\right]d\varphi = \left|\begin{array}{ll} 1+4r^2=t^2 & r=0 \Rightarrow t=1 \\ 8r\,dr=2t\,dt & r=\sqrt{2} \Rightarrow t=3 \\ r\,dr=\frac{1}{4}t\,dt & \end{array}\right| =$$

$$= 6\int_0^{2\pi}\left[\int_1^3 \frac{1}{4}t^2\,dt\right]d\varphi = 6\cdot\frac{1}{4}\,\varphi\Big|_0^{2\pi}\cdot\frac{t^3}{3}\Big|_1^3 = \frac{3}{2}\cdot\frac{1}{3}\cdot 2\pi\cdot 26 = 26\pi$$

$$3\iint\limits_{D} r^3\sqrt{1+4r^2}\,dr\,d\varphi = 3\int_0^{2\pi}\left[\int_0^{\sqrt{2}} \underbrace{r^3}_{r^2\cdot r}\sqrt{1+4r^2}\,dr\right]d\varphi = \left|\begin{array}{ll} 1+4r^2=t^2 & r\,dr=\frac{1}{4}t\,dt \\ 4r^2=t^2-1 & r=0 \Rightarrow t=1 \\ r^2=\frac{1}{4}(t^2-1) & r=\sqrt{2} \Rightarrow t=3 \\ 8r\,dr=2t\,dt & \end{array}\right| =$$

$$= 3\int_0^{2\pi}\left[\int_1^3 \frac{1}{16}(t^2-1)\,t\cdot t\,dt\right]d\varphi = \frac{3}{16}\int_0^{2\pi}\left[\int_1^3 (t^4-t^2)\,dt\right]d\varphi = \frac{3}{16}\cdot\varphi\Big|_0^{2\pi}\cdot\left(\frac{1}{5}t^5\Big|_1^3 - \frac{1}{3}t^3\Big|_1^3\right)$$

$$= \frac{3}{8}\pi\cdot\left(\frac{242}{5} - \frac{26}{3}\right) = \frac{1}{8}\pi\left(\frac{726}{5} - 26\right) = \frac{1}{8}\pi\,\frac{726-130}{5} = \frac{596\pi}{40} = \frac{149\pi}{10}$$

$$\iint\limits_{S} 3z\,dS = 26\pi - \frac{149\pi}{10} = \frac{260-149}{10}\pi = \frac{111\pi}{10}$$ traženo rješenje